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# KAPUTO MA'NOSIDAGI TEMPERLANGAN KASR TATRIBLI TENGLAMA UCHUN KOSHI MASALASI

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**Abstract:** Ushbu maqolada Kaputo ma'nosidagi temperlangan kasr tartibli integro-differensial operator qatnashgan differensial tenglama uchun boshlang'ich shartli masalaning yechimi yaqqol ko'rinishda topilgan.

**Keywords:** Kasr taribli operatorlar, Kaputo ma'nosidagi temperlangan kasr tartibli operator, Riman-Liuvill temperlangan integral operatori, integral tenglama.

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Ma'lumki oxirgi yillarda kasr tartibli differensial tenglamalarga qiziqish tobora ortib bormoqda. Bu bir tomondan, matematik umumlashma sifatida oldingi natijalarni o'z ichiga olsa, boshqa tomondan ko'pgina amaliy jarayonlarning matematik modelida turli kasr tartibli differensial tenglamalar ishlatilmoqda [1,2]. Bunday differensial tenglamalarni tadqiq etishda boshlang'ich shartli masalaning yaqqol ko'rinishdagi yechimlari muhim rol o'ynaydi [3]. Bunday masalalarni tadqiq etishda Laplas almashtirishlari, operator usullar yoki integral tenglamalarga keltirib ishlash usullaridan foydalaniladi [4].

Kasr tartibli operatorlarning asosiylari Riman-Liuvill va Kaputo ma'nosidagi operatorlar bo'lsa, ularning turli umumlashmalari ko'p ishlarda tadqiq etilmoqda [5].

Ushbu ishda temperlangan Kaputo ma'nosidagi kasr tartibli operator [6] qatnashgan differensial tenglama uchun Koshi masalasini tadqiq etamiz. Bu operatordan parametrning ma'lum qiymatida Kaputo ma'nosidagi kasr tartibli operator kelib chiqadi va olingan natijalar mos ravishda umumlashma xarakterga ega bo'ladi.

Bunday tenglama uchun Koshi masalasi, tenglamaning o'ng tomoni yechimga bog'liq bo'lgan holatda [7] da tadqiq etilgan. Shuni ta'kidlash zarurki, bu ishda Koshi masalasing yechimi yaqqol ko'rinishda topilmagan. Biz esa chiziqli tenglama uchun Koshi masalasi yechimini yaqqol ko'rinishda topamiz.

$$ {}^{c}D_{0t}^{\alpha,\lambda} y(t) - \mu y(t) = f(t), t > 0 \qquad (1) $$

tenglamani qaraymiz , bu yerda $0<\alpha<1, \lambda\geq 0, \mu\in R$,

$${}^{c}D_{0t}^{\alpha,\lambda}y(t)=\frac{e^{-\lambda t}}{\Gamma(1-\alpha)}\int_{0}^{t}(t-s)\frac{d}{ds}[e^{\lambda s}y(s)]ds$$

-Kaputo ma'nosidagi temperlangan $0<\alpha<1$ kasr tartibli integro-differensial operator [6]. (1) tenglamaning umumiy yechimini topish uchun tenglamaning ikkala tomoniga Riman-Liuvill temperlangan kasr tartibli integral operatori

$$I_{0t}^{\alpha,\lambda}x(t)=\frac{1}{\Gamma(\alpha)}\int_{0}^{t}e^{-\lambda(t-s)}(t-s)^{\alpha-1}x(s)ds \qquad (2)$$

ni ta'sir ettiramiz:

$$I_{0t}^{\alpha,\lambda c}D_{0t}^{\alpha,\lambda}y(t)-\mu I_{0t}^{\alpha,\lambda}y(t)=I_{0t}^{\alpha,\lambda}f(t). \qquad (3)$$

Bu tenglamadan

$$I_{0t}^{\alpha,\lambda c}D_{0t}^{\alpha,\lambda}y(t)=y(t)-y(0)e^{-\lambda t}$$

tenglikni hisobga olib, (3) tenglamani quyidagi ko'rinishda yozamiz:

$$y(t)-\mu I_{0t}^{\alpha,\lambda}y(t)=I_{0t}^{\alpha,\lambda}f(t)+y(0)e^{-\lambda t},$$

bu yerda ushbu $g(t)=I_{0t}^{\alpha,\lambda}f(t)+y(0)e^{-\lambda t}$ belgilashni kiritsak,

$$y(t)-\mu I_{0t}^{\alpha,\lambda}y(t)=g(t) \qquad (4)$$

integral tenglamaga kelamiz. Agar biz (1) tenglamani $y(0)=y_0, y_0\in$ boshlang'ich shart bilan birgalikda qarasak, bu Koshi masalasi (4) integral tenglamaga ekvivalent bo'ladi. Ushbu integral tenglamani yechish uchun ketma-ket yaqinlashish usulidan foydalanamiz:

$$y_0(t)=g(t); y_1(t)=g(t)+\mu I_{0t}^{\alpha,\lambda}g(t); y_2(t)=g(t)+\mu I_{0t}^{\alpha,\lambda}y_1(t)$$

yoki

$$y_2(t)=g(t)+\mu I_{0t}^{\alpha,\lambda}g(t)+\mu^2 I_{0t}^{\alpha,\lambda}I_{0t}^{\alpha,\lambda}g(t),$$

bu yerda $I_{0t}^{\alpha,\lambda}I_{0t}^{\beta,\lambda}g(t)=I_{0t}^{\alpha+\beta,\lambda}$ xossaga ko'ra

$$y_2(t)=g(t)+\mu I_{0t}^{\alpha,\lambda}g(t)+\mu^2 I_{0t}^{2\alpha,\lambda}g(t)$$

ko'rinishda yoziladi. Keyingi qadamda

$$y_3(t)=g(t)=\mu I_{0t}^{\alpha,\lambda}y_2(t)$$

ifodani $y_2(t)$ ni o'rniga odingi ko'rinishni qo'yib,

$$y_3(t)=g(t)=\mu I_{ot}^{\alpha,\lambda}g(t)+\mu^2 I_{0t}^{2\alpha,\lambda}+\mu^3 I_{0t}^{3\alpha,\lambda}g(t)$$

tenglikni hosil qilamiz. Bu jarayonni davom ettirib,

$$y_n(t)=g(t)+\sum_{k=1}^{n}\mu^k I_{ot}^{k\alpha,\lambda}g(t)$$

tenglikka ega bo'lamiz va $n\to\infty$ da limitga o'tib (4) tenglamaning yechimini ushbu

$$y(t) = g(t) + \sum_{k=1}^{\infty} \mu^k I_{0t}^{k\alpha,\lambda} g(t) \qquad (5)$$

qator ko‘rinishda hosil qilamiz. (5) ni soddalashtirish uchun (2) ifodadan foydalanamiz:

$$y(t) = g(t) + \sum_{k=1}^{\infty} \frac{\mu^k}{\Gamma(k\alpha)} \int_0^t e^{-\lambda(t-s)} (t-s)^{k\alpha-1} g(s) ds.$$

$k = n+1$ almashtirish natijasida quyidagini olamiz:

$$y(t) = g(t) + \sum_{n=0}^{\infty} \frac{\mu^{n+1}}{\Gamma(n\alpha+\alpha)} \int_0^t e^{-\lambda(t-s)} (t-s)^{n\alpha+\alpha-1} g(s) ds.$$

Endi $E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)}$ ekanligidan

$$E_{\alpha,\alpha}(\mu(t-s)^\alpha) = \sum_{n=0}^{\infty} \frac{\mu^n (t-s)^{\alpha n}}{\Gamma(\alpha n+\alpha)}$$

kelib chiqadi [4] va natijada yechimning quyidagi ko‘rinishini olamiz:

$$y(t) = g(t) + \mu \int_0^t e^{-\lambda(t-s)} (t-s)^{\alpha-1} E_{\alpha,\alpha}(\mu(t-s)^\alpha) g(s) ds.$$

Bu yerdagi $g(t)$ ni o‘rniga $g(t) = I_{0t}^{\alpha,\lambda} f(t) + y_0 e^{-\lambda t}$ belgilashni qo‘ysak,

$$y(t) = I_{0t}^{\alpha,\lambda} f(t) + y_0 e^{-\lambda t} + \mu \int_o^t e^{-\lambda(t-s)} (t-s)^{\alpha-1} E_{\alpha,\alpha}(\mu(t-s)) ds.$$

hosil bo‘ladi va ifodani soddalashtirsak,

$$y(t) = y_0 e^{-\lambda t} + \mu y_0 e^{-\lambda t} t^\alpha E_{\alpha,\alpha+1}(\mu t^\alpha) + \int_0^t e^{-\lambda(t-\tau)} (t-\tau)^{\alpha-1} E_{\alpha,\alpha}(\mu(t-\tau)) f(\tau) d\tau$$

ko‘rinishga keladi.

$$E_{\alpha,\beta}(z) - z E_{\alpha,\alpha+\beta}(z) = \frac{1}{\Gamma(\beta)}$$

xossadan [3] foydalanish yechimni quyidagi yakuniy ko‘rinishga keltiradi:

$$y(t) = y_0 e^{-\lambda t} E_\alpha(\mu t^\alpha) + \int_0^t e^{-\lambda(t-\tau)} (t-\tau)^{\alpha-1} E_{\alpha,\alpha}(\mu(t-\tau)^\alpha) f(\tau) d\tau. \qquad (6)$$

Demak, quyidagi tasdiq o‘rinli bo‘ladi:

**Teorema**. *Agar $f(t)$ funksiya uzluksiz differensiallanuvchi bo‘lsa, u holda (1) tenglamaning $y(0) = y_0$ shartni qanoatlantiruvchi yechimi mavjud, yagona va u (6) ko‘rinishda aniqlanadi.*

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