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PARABOLIK–GIPERBOLIK TIPDAGI TENGLAMALAR UCHUN XARAKTERISTIKADAN SILJIGAN CHIZIQLARNI O`Z ICHIGA OLGAN QUYI YARIM SOHADA CHEGARAVIY MASALA

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Anotatsiya.

Ushbu maqolada parabolik–giperbolik tipdagi tenglama uchun xarakteristikadan siljigan chiziqlarni o`z ichiga olgan quyi yarim sohalarda chegaraviy masala uchun qo'yilgan nolokal shartli chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan.

Kalit so'zlar.

Parabolik–giperbolik tipdagi tenglama, xarakteristik uchburchak, regulyar yechim, integral energiya usuli, trivial yechim, Grin funksiyasi, Volterraning ikkinchi tur integral tenglamasi, Dalamber formulasi.

КРАЕВАЯ ЗАДАЧА В НИЖНИХ ПОЛУПОЛЯХ, ОТКЛОНЯЮЩИХСЯ ОТ ХАРАКТЕРИСТИК ДЛЯ УРАВНЕНИЯ ПАРАБОЛО – ГИПЕРБОЛИЧЕСКОГО ТИПА

Аннотация.

В данной статье доказывается существование и единственность решения нелокальной условной краевой задачи для краевой задачи в нижних полуполях, содержащих линии, сдвинутые с характеристики, для уравнения параболо-гиперболического типа.

Ключевые слова.

Уравнение параболо-гиперболического типа, характеристический треугольник, обычное решение, метод интегральной энергии, простое решение, функция Грина, Интегральное уравнение Вольтерра второго рода, Формула Даламбера.

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BOUNDARY VALUE PROBLEM IN LOWER SEMIFIELDS DEVIATING FROM CHARACTERISTICS FOR A PARABOLIC–HYPERBOLIC EQUATION

Annotation.

This article proves the existence and uniqueness of a solution to a nonlocal conditional boundary value problem for a boundary value problem in lower semifields containing lines shifted from the characteristic for an equation of parabolic-hyperbolic type.

Key words.

Equation of parabolic-hyperbolic type, characteristic triangle, usual solution, integral energy method, simple solution, Green's function, Volterra integral equation of the second kind, Dalembert's formula.

1. Masalaning qo`yilishi

Quyidagi tenglamani qaraymiz:

$$0 = Lu \equiv \begin{cases} u_{xx} - u_y, & (x,y) \in \Omega_0, \\ u_{xx} - u_{yy}, & (x,y) \in \Omega_j \left(j = \overline{1,3} \right), \end{cases} \tag{1}$$

bu yerda Ω_0 soha deb $x>0,\ y>0$ bo`lganda $y=0,\quad x=1,\quad y=1,\quad x=0$ to`g′ri chiziqlarda mos ravishda joylashgan $AB,\ BB_0,\ B_0A_0,\ A_0A,$ kesmalar bilan chegaralangan to`rtburchak sohani, Ω_1 soha $x<0,\ y>0$ da ΔAA_0D xarakteristik uchburchakning ichida joylashgan $AK: x=\gamma_1(y)$ silliq egri chiziq va (1) tenglamanig $BP:\ y-x=1$ xarakteristikasi bilan chegaralangan soha, Ω_2 soha $x>0,\ y>0$ da ΔBB_0E xarakteristik uchburchakning ichida joylashgan $AC: x=-\gamma_2(y)$ silliq egri chiziq va (1) tenglamanig $B_0M:\ x+y=2$ xarakteristikasi bilan chegaralangan soha, Ω_3 soha $x<0,\ y<0$ da (1) tenglamanig $AC:\ y+x=0$ va $BC:\ y-x=-1$ xarakteristikasi bilan chegaralangan ΔAA_0D xarakteristik uchburchak soha,

Quyidagi belgilashlarni kiritamiz: $J_1=\{(x,y):\ 0<x<1,\ y=0\}$,

$J_2=\{(x,y):\ x=0, 0<y<1\}$, $J_3=\{(x,y):\ x=1,\ 0<y<1\}$,

$\Omega=\Omega_0\cup\Omega_1\cup\Omega_2\cup\Omega_3\cup J_1\cup J_2\cup J_3,\ C\left(-\frac{1}{2},\frac{1}{2}\right),\quad D\left(-\frac{1}{2},\frac{1}{2}\right),\quad E\left(\frac{3}{2},\frac{1}{2}\right),$

$\theta\left(\frac{x}{2};-\frac{x}{2}\right),\ \left[\theta^*\left(\frac{\lambda(x)+x}{2};\frac{\lambda(x)-x}{2}\right)\right]$

bu yerda $\theta_2(x)\left[\theta_2^*(x)\right]$ (1) tenglamaning $(x,0)\in J_3$ nuqtadan chiquvchi xarakteristikalari bilan $AC[AN]$ xarakteristikalarining kesishish nuqtasining koordinatalari.

A-Shart. $y=-\gamma_1(x)$ va $\gamma_j(y)(j=2,3)$ - berilgan funksiyalar bo`lib quyidagi shatlarni bajarsin:

1) $\gamma_1(x)$ va $\gamma_j(y)(j=2,3)$ funksiyalar mos ravishda ΔACB va ΔAA_0D, ΔBEB_0 xarakteristik uchburchaklar ichidaqa to`liq joylashgan bo`lsin;

2) $\gamma_1(x)\in C^2(0,1)$, $\gamma_j(y)\in C^2(0,1)(j=2,3)$ tegishli bo`lsin;

3) $t\pm\gamma_j(t)\left(j=\overline{1,3}\right)$ - monoton o'suvchi;

4) $\gamma_1(0)=0,\ \gamma_2(0)=-1,\ l_1+\gamma_1(l_1)=1,\ l_2-\gamma_2(l_2)=2, l_3+\gamma_3(l_3)=1,\ l_j=const$ $l_j\in\left(\frac{1}{2};1\right)$.

Ta'rif. (1) tenglamaning Ω sohadagi regulyar yechimi deb,

$W_1=\left\{u:u(x,y)\in C^1\left(\overline{\Omega}\right)\cap C_{x,y}^{2,1}\left(\Omega_0\right)\cap C^2\left(\Omega_i\right), i=\overline{1,3}\right\}$

sinfga tegishli bo'lgan hamda $\Omega_i\left(i=\overline{0,3}\right)$ sohada (1) tenglamani qanoatlantiruvchi $u(x,y)$ yechimga aytiladi.

I – Masala. Quyidagi shartlarni qanoatlantiruvchi (1) tenglamaning regulyar yechimi topilsin:

$$\left[u_x-u_y\right]\theta(x)+\mu(x)\left[u_x-u_y\right]\theta^*(x)=\varphi(x); \qquad (2)$$

$$u|_{A_0D}=g_1(y);\ u|_{B_0E}=g_2(y) \qquad (3)$$

$$\left[u_x+u_y\right]\Big|_{AD}=p(y); \qquad (4)$$

$$\left[u_x-u_y\right]\Big|_{BE}=q(y); \qquad (5)$$

$$u(A)=u(B)=0; \qquad (6)$$

Bunda $\mu(x)$, $\varphi(x)$, $g(y)$, $p(y)$ va $q(y)$ - berilgan yetarlicha silliq funksiyalar.

Teorema. Agar $\mu(x)\neq-1;\ \mu(x),\varphi(x)\in C^1[0,1]\left(i=\overline{1,3}\right), g(y),\ p(y),$ $q(y)\in C^2(0;1)$ va **A** shartlar bajarilsa, I - masalaning yagona regulyar yechimi mavjud bo'ladi.

Isbot. Ω_1 sohalarda Koshi masalasining yechimining ko'rinishi quyidagicha bo'ladi.

$$u(x,y)=\frac{1}{2}\left[\tau_1(x+y)+\tau_1(x-y)+\int_{x-y}^{x+y}\nu_1(t)dt\right],\ 0<x<1$$

Bundan $\theta\left(\frac{x}{2};-\frac{x}{2}\right)$va $\theta^*\left(\frac{\lambda(x)+x}{2};\frac{\lambda(x)-x}{2}\right)$ nuqtalarda quyidagi funksional munosabatlarni olamiz:

$$(1+\mu(x))\nu_1(x)=[1+\mu(x)]\tau_1'(x)-\varphi(x),\ 0<x<1 \qquad (7)$$

(1) dan Ω_0 sohada $y\to+0$ limitga o'tib quyidagi tenglamani olamiz.

$$(\tau_1(x))''=\nu_1(x) \qquad (8)$$

(7) ni (8) ga qo'yib $\tau_1(x)$ga nisbatan ikkinchi tartibli oddiy differensial tenglamani hosil qilishimiz mumkin.

$$(\tau_1(x))''-\tau_1'(x)=-\frac{\varphi(x)}{1+\mu(x)} \qquad (9)$$

(9) tenglamani $\tau_1(0)=0$ va $\tau_1(1)=0$ shartlar ostida yechimning umumiy ko'rinishining shakli tasvirlanadi.

$$\tau_1(x)=\int_0^x\frac{\varphi(t)\left[1-e^{x-t}\right]}{1+\mu(t)}dt+\frac{e^x-1}{e-1}\int_0^1\frac{\varphi(t)\left[e^{1-t}-1\right]}{1+\mu(t)}dt \qquad (10)$$

(8) dan foydalanib, $\nu_1(x)$ ni topamiz

$$\nu_1(x)=-\int_0^x\frac{\varphi(t)e^{x-t}}{1+\mu(t)}dt+\frac{e^x}{e-1}\int_0^1\frac{\varphi(t)\left(e^{1-t}-1\right)}{1+\mu(t)}dt-\frac{\varphi(x)}{1+\mu(x)} \qquad (11)$$

Integral tenglamalar metodi bilan masalaning yechimini mavjudligini isbotlaymiz. Buning uchun biz (8)- (9) funksional munosabatlardan va $\Omega_0$sohada (1) tenglama uchun qo`yilgan birinchi chegaraviy masalaning yechimidan

$$u(x,y)=\int_0^1\tau_1(t)G(x,y;t,0)dt+\int_0^y\tau_2(t)G_t(x,y;0,z)dz-$$
$$-\int_0^y\tau_3(z)G_t(0,y;1,z)dz, \qquad (12)$$

ko'rinishda bo'ladi.

Bu yerda $G(x,y;t,z)=\frac{1}{2\sqrt{\pi(y-z)}}\sum_{n=-\infty}^{\infty}\left[e^{-\frac{(x-t+2n)}{4(y-z)}}-e^{-\frac{(x+t+2n)}{4(y-z)}}\right]$ - issiqlik o'tkazuvchanlik tenglamasi uchun birinchi chegaraviy masalaning Grin funksiyasi.

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$\tau_k(y)$, $\nu_k(y)$ $(k=2,3)$ funksiyalar orasidagi munosabat olish uchun bir marta x bo'yicha differensiallab:

$$u_x(x,y)=\int_0^1\tau_1(t)G_x(x,y;t,0)dt+\int_0^y\tau_2(z)G_{tx}(x,y;0,z)dz-$$
$$-\int_0^y\tau_3(z)G_{tx}(x,y;1,z)dz, \quad (13)$$

ni hosil qilamiz. Quyidagi

$$N(x,y;t,z)=\frac{1}{2\sqrt{\pi(y-z)}}\sum_{n=-\infty}^{\infty}\left[e^{-\frac{(x-t+2n)^2}{4(y-z)}}+e^{-\frac{(x+t+2n)^2}{4(y-z)}}\right]$$ belgilash kiritsak,

$G_{tx}(x,y;t,z)=N_z(x,y;t,z)$, $G_x(x,y;t,z)=-N_t(x,y;t,z)$ munosabatlarga ega bo'lamiz. $u_x(0,y)=\nu_2(y)$ belgilashga ko'ra (13) dan quyidagi munosabatlarni olamiz:

$$\nu_2(y)=\int_0^1\tau_3'(z)N(x,y;1,z)dt-\int_0^y\tau_1'(t)N(x,y;t,0)dt+\int_0^y\tau_2'(z)N(0,y;0,z)dz$$
$$\nu_3(y)=\int_0^1\tau_3'(z)N(1,y;1,z)dz-\int_0^y\tau_1'(z)N(1,y;t,0)dz+\int_0^y\tau_2'(z)N(1,y;0,z)dz \quad (14)$$

Ma'lumki, $u_{xx}-u_{yy}=0$ tenglamaning umumiy yechimi

$$u(x,y)=f_1(x+y)+f_2(x-y) \quad (15)$$

ko'rinishda bo'ladi, bunda $f_1(.)$, $f_2(.)$ - ikkinchi tartibli uzluksiz differensiallanuv – chi noma'lum funksiya.

(4) shartidan va (15) dan $f_1'(y-\gamma_2(y))=p(y)$, $0\le y\le l$ ga ega bo'lamiz, tenglamadan $y-\gamma_2(y)=t$ ni yechini $y=\delta_1(t)$ ko'rinishda izlab

$$f_1'(t)=\frac{1}{2}p(\delta_1(t)),\ 0\le y\le l,$$ bundan

$$f_1(y)=f_1(0)+\frac{1}{2}\int_0^y p(\delta(t))dt,\ 0\le y\le l.$$

(3.8) shartidan va (15) dan $f_2'(y-\gamma_2(y))=q(y)$, $0\le y\le l$ ga ega bo'lamiz, tenglamadan $y-\gamma_2(y)=t$ ni yechini $y=\delta_1(t)$ ko'rinishda izlab

$$f_2'(t)=\frac{1}{2}q(\delta_1(t)),\ 0\le y\le l,$$ bundan

$$f_2(y)=f_2(0)+\frac{1}{2}\int_0^y q(\delta(t))dt,\ 0\le y\le l.$$

Endi $l \le y \le 1$ da $u|_{A_0D} = g_1(y)$ va $u|_{B_0E} = g_2(y)$ shartni hisobga olsak,

$$\begin{cases} f_1(y) = g_1\left(\dfrac{y-1}{2}\right) + f_2(1),\ l \le y \le 1 \\ f_2(y) = g_2\left(\dfrac{2-y}{2}\right) + f_1(2),\ l \le y \le 1 \end{cases}$$

(15) ga $f_1(y)$ va $f_2(y)$ ning qiymatni qo'yamiz va quyidagiga ega bo'lamiz.

$$u(x,y) = \begin{cases} f_2(x-y) + \dfrac{1}{2}\int\limits_0^y p(\delta(t))dt + f_1(0), 0 \le y \le l, \\ f_2(x-y) + g_1\left(\dfrac{y-1}{2}\right) + f_2(1),\ l \le y \le 1. \\ f_1(x+y) + \dfrac{1}{2}\int\limits_0^y q(\delta(t))dt + f_2(0), 0 \le y \le l, \\ f_1(x+y) + g_2\left(\dfrac{2-y}{2}\right) - f_1(2),\ l \le y \le 1. \end{cases} \qquad (16)$$

$u_y(0,y) = \tau_i'(y),\ i = 2,3$ ligidan, (16) tenglikni y bo'yicha bir marta differensiallab $x \to 0$ desak

$$\tau_2'(y) = \begin{cases} -f_2'(-y) + \dfrac{1}{2}p(\delta(y)), \quad 0 \le y \le l, \\ -f_2'(-y) + \dfrac{1}{2}g_1'\left(\dfrac{y-1}{2}\right),\ l \le y \le 1. \end{cases} \qquad (17)$$

$x \to 1$ da esa

$$\tau_3'(y) = \begin{cases} f_1'(1+y) + \dfrac{1}{2}q(\delta(1+y)), \quad 0 \le y \le l, \\ f_1'(1+y) - \dfrac{1}{2}g_2'\left(\dfrac{1-y}{2}\right),\ l \le y \le 1. \end{cases} \qquad (18)$$

(16) ni (17)va (18) ga qo'yib $f_i'(y)$ $i = 2,3$funksiya uchun $\tau_i(y)$ $i = 2,3$ va $\nu_i(y)$ $i = 2,3$funksiyalar o'rtasida quyidagi funksional munosabatni olamiz:

$$\begin{cases} \tau_2'(y) = \nu_2(y) + p(\delta(y)), \quad 0 \le y \le l,\ x < 0 \\ \tau_2'(y) = \nu_2(y) + g_1'\left(\dfrac{y-1}{2}\right),\ l \le y \le 1.\ x < 0 \end{cases} \qquad (19)$$

$$\begin{cases} \tau_3'(y) = \nu_3(y) + q(\delta(1-y)), 0 \le y \le l,\ x > 1 \\ \tau_3'(y) = \nu_3(y) + g_2'\left(\dfrac{1-y}{2}\right),\ l \le y \le 1, \quad x > 1 \end{cases} \qquad (20)$$

(19)va (20) dan $\nu_2(y)$ va$\nu_3(y)$ ni (14) ga qo'ysak

$$\tau_2'(y)-\int_0^1\tau_3'(z)N(x,y;1,z)dt+\int_0^y\tau_1'(t)N(x,y;t,0)dt-\int_0^y\tau_2'(z)N(0,y;0,z)dz=F_2(y)$$

$$\tau_3'(y)-\int_0^1\tau_3'(z)N(1,y;1,z)dz+\int_0^y\tau_1'(z)N(1,y;t,0)dz-\int_0^y\tau_2'(z)N(1,y;0,z)dz=F_3(y)$$

(21)

Bu yerda

$$F_2(y)=\frac{1}{2}p(\delta(1-y))-\frac{1}{2}g_1'\left(\frac{1-y}{2}\right)$$

$$F_3(y)=\frac{1}{2}q(\delta(1-y))-\frac{1}{2}g_2'\left(\frac{1-y}{2}\right)$$

(21) sistemani

$$\begin{cases}\tau_2'(y)-\int_0^y\tau_2'(z)N(0,y;0,z)dz=F_2^*(y),\\ \tau_3'(y)+\int_0^y\tau_3'(z)N(1,y;1,z)dz=F_3^*(y).\end{cases}\qquad(22)$$

ko'rinishda olamiz, bunda

$$\begin{cases}F_2^*(y)=F_2(y)+\int_0^y\tau_3'(z)N(x,y;1,z)dz-\int_0^1\tau_1'(t)N(x,y,t,0)dt,\\ F_3^*(y)=F_3(y)+\int_0^y\tau_2'(z)N(1,y;0,z)dz-\int_0^y\tau_1'(z)N(1,y,t,0)dt\end{cases}\qquad(23)$$

(14) sistema $\left|N(0,y,0,z)\right|\le\frac{2}{\sqrt{\pi|y-y_1|}}\sum_{n=1}^{\infty}\left|e^{-\frac{n^2}{|y-y_1|}}\right|\le const$

$\left|F_2^*(y)\right|\le const$ bo'lgani uchun (23) sistemadan 1 – tenglamani ketma – ket yaqinlashish usuli bilan yechib

$$\tau_2(y)=F_2^*(y)+\int_0^y F_2^*(t)K(t,y)dt\qquad(24)$$

ni olamiz.

$F_2^*(y)$ ni (24) ga qo'yib

$$\tau_2(y)=F_2(y)+\int_0^y\tau_3'(z)N(x,y;1,z)dz-\int_0^1\tau_1'(t)N(x,y,t,0)dt+$$
$$+\int_0^y\left(F_2(y)+\int_0^t\tau_3'(z)N(x,y;1,z)dz-\int_0^y\int_0^1\tau_1'(p)N(x,y,p,0)dp\right)K(t,y)dt\qquad(25)$$

Va nihoyat (25) ni (21) ning 2 – tenglamasiga qo'yamiz va $\tau_2(y)$ ga nisbatan Volterra ikkinchi tur integral tenglamasini hosil qilamiz:

$$\tau_3'(y)-\int_0^y \tau_3'(z)N(1,y;1,z)dz+\int_0^y \tau_1'(z)N(1,y;t,0)dz-\int_0^z N(1,y;0,z)dz-\int_0^y \tau_3'(z)N(x,y;1,z)+$$
$$+\int_0^y \tau_3'(z)N(x,y;1,z)dz-\int_0^y\int_0^1 \tau_1'(p)N(x,y,p,0)dpK(t,y)dp+F_3(y)=F_2(y) \quad (26)$$

bu yerda

$$F_3(y)=\int_0^y F_2(z)N(1,y,0,z)dz+\int_0^y F_2(s)K(s,y)ds+\int_0^y N(1,y,0,z)\int_0^y F_2(p)K(p,y)dp \quad (27)$$

(26) tenglamani yechib $\tau_2(y)$, bundan va (25) dan $\tau_1(y)$va (26), (27)dan $\nu_1(y)$va $\nu_2(y)$ni topamiz. $\tau_i(y),\nu_i(y)\left(i=\overline{1,3}\right)$ lar ma'lum funksiyalar. Endi Ω_0 sohada I masalaning yechimini tiklashimiz mumkin, $\overline{\Omega_i}\left(i=\overline{1,3}\right)$ sohalarda esa Koshi masalasining yechimi bo'lgan Dalamber formulasi orqali yechim topiladi. Demak, I-masala bir qiymatli yechildi.

Teorema isbotlanadi.

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