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STATISTICAL ANALYSIS OF CURRENT PEDAGOGICAL EXPERIMENTS USING CONTROL CHARTS

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Abstract.

Purpose *- The purpose of this article is to reduce the statistical analysis of the pedagogical experiment to diagrams using control charts.*

Design/methodology/approach*. The study uses an inductive method, while decisions are made based on current real data.*

Findings. *Research shows that the constructed new control cs not only check the homogeneity of two samples, they can also be used as a tool for checking the quality of the educational process, as a document of the experiments conducted, a common language of various specialists and as the voice of the pedagogical process.*

Research limitations/implications*. The results of the work can be used to study other processes where checking the uniformity of two samples is required.*

Practical implications*. The methodology used can be used to improve the pedagogical experiment and to compare the same experiments given by other researchers.*

Originality/value*. The research fills the gaps in the analysis of pedagogical experiments by statistical methods.*

Keywords.

Smirnov's homogeneity criterion, Pearson's homogeneity criterion, control charts to control the heterogeneity of two independent samples, control chart boundaries, analysis of the pedagogical experiment using control charts, experimental and control group.

Introduction.

Currently, most studies in the study of various processes use a statistical tool, the so-called Shewhart Control Chart (CCS) [1,2,3,4]. The study of education problems can be found, for example, in the following works [3,5,6,14,19]. Therefore, in a number of situations in the analysis of pedagogical data, there is a need to move from classical methods to the CC method. In this article, as an example, we will consider the problem of checking the homogeneity of two samples:

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$x_1, x_2, \dots, x_m$ and $y_1, y_2, \dots, y_n$ (i.e. sets of m and n real numbers) - it is required to check if there is a difference between the samples. Using the Smirnov method and Pearson's chi-squared, we will build a Control Chart to test the homogeneity [7,8,9.13].

Qualitative and quantitative analysis of pedagogical phenomena and processes by methods of probability theory and mathematical statistics clarify the methods and hypotheses put forward. Usually, statistical conclusions are made after the end of the experiments. Then we have results like "Off line control" [8,10,11,12,15,16,17,18,19].

In practice, the results of the "On line control" type are important, i.e. the obtained statistical conclusions are used during the experiment. These results simultaneously make it possible to verify the correctness of the experiment (method) carried out and to correct shortcomings in the experimental plans in a timely manner. This procedure is called SPC (Statistical Process Control). SPC uses various statistical tools, in particular the CC. Usually the CC is called the "voice of the process". It is a universal language for different professionals using statistical inference in their research. For each CC, there is a corresponding statistical hypothesis and a test to test that hypothesis.

The Shewhart CC for alternative data (p, np, c, u) can be used to analyze and evaluate the effectiveness and efficiency of the educational process [3]. The Shewhart CC for quantitative data ($\overline{X}, S, m, R$) can be used to determine the monitoring of the learning process.

If the researcher has started to act according to the planned procedure to improve the learning process (new teaching methods, changed learning content, correct repetition of educational material, some new way of remembering, new curriculum ...), then statistical control of the process is required. We believe that the use of the CC method is productive here.

Purpose and methods of research.

For statistical analysis of current pedagogical experiments using the CC method, you first need to find the characteristics of the CC used.

- control value related to the quality of the educational process;
- control limits: LCL - lower control limit; UCL - upper control limit.

There are two approaches to building a CC:

Probabilistic Approach: Here the CC performs the task of sequentially testing certain statistical hypotheses. To find the control limits, knowledge of the distribution of the control value (exact or limiting) and a 99% confidence interval are required [2].

Operational Approach: As in the first approach, the Control Chart performs the task of sequentially testing certain statistical hypotheses. To find the control limits, knowledge of the distribution of the control value is not required; they are found according to the "six sigma" rule [2,4].

In this article, we use the first approach when constructing the CC. And as the criteria used, we take the criterion of homogeneity of two samples of Smirnov [10] and the chi-square test of K. Pearson [7] to test the put forward hypotheses.

Here we briefly outline the classical method for testing homogeneity and using these criteria for the study of pedagogical processes. Then, based on them, we find suitable characteristics of the new Control Chart used.

Smirnov's test for checking the homogeneity of two samples.

Let samples be taken from the general populations X and Y with distribution functions $F(x)$ and $G(x)$, and they are ordered, respectively, in ascending order: $x_1 < x_2 < \cdots < x_m$, $y_1 < y_2 < \cdots < y_n$. Let us denote the empirical distributions of X and Y as $F_m(x)$ and $G_n(x)$, respectively. Let's put $d = max\{G_n(x) - F_m(x)\}$.

The Smirnov test statistic $\lambda = d\sqrt{\frac{nm}{n+m}}$ measures the difference between the empirical distribution functions built from samples, and in the limit obeys the Kolmogorov distribution [13].

With limited m and n, the random variable d is discrete, and with the help of λ, the following hypotheses related to pedagogical processes can be tested:

$H_0 : F(x) = G(x)$ for all x;

$H_1 : F(x) < G(x)$for at least one x.

Smirnov's criterion for $n; m \geq 50$ allows you to find the point at which the sum of the accumulated differences between the two distributions is the largest and evaluate the reliability of this discrepancy.

In particular, we will consider here the following typical problem in pedagogical research.

Let the distribution of objects of two collections (experimental and control classes) be compared according to the state of some property (for example, the performance of a certain task). At the same time, students are divided into four categories in accordance with the marks (in points 2; 3; 4; 5) received for the performance of some control work. It is required to test the hypotheses H_0 and H_1 at a significance level of α.

According to the rule, homogeneity is rejected and the hypothesis H_1 is accepted if $\lambda > k_{1-\alpha}$, where$k_{1-\alpha}$ is the quantile of the Kolmogorov distribution.

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Chi-square test for testing the homogeneity of two samples.

The chi-square test of homogeneity is applicable to the analysis of data of any nature and any finite number of samples can be analyzed simultaneously.

Suppose two series of independent observations were made from the control and experimental groups consisting of n_1 and n_2 observations, respectively $\underline{X} = (x_1, x_2, \dots, x_{n_1})$ and $\underline{Y} = (y_1, y_2, \dots, y_{n_2})$. Moreover, the measured property has $N \geq 2$ categories. Further, in accordance with specially developed criteria, we determine the frequency of hitting the elements of the samples in one of the N categories. Let $\underline{v}_1 = (v_{11}, v_{12}, \dots, v_{1N})$, $\underline{v}_2 = (v_{21}, v_{22}, \dots, v_{2N})$ outcome frequencies, and $\underline{p}_1 = (p_{11}, p_{12}, \dots, p_{1N})$, $\underline{p}_2 = (p_{21}, p_{22}, \dots, p_{2N})$- probabilities of the control and experimental groups. Then the hypothesis of homogeneity, meaning the statement that the probabilities of outcomes did not change in the control and experimental groups, read:

$$H_0: \underline{p}_1 = \underline{p}_2 = \underline{p} \qquad (1)$$

where $p = (p_1, p_2, \dots, p_N)$, $(p_1 + p_2 + \dots + p_N = 1)$ is an unknown probability vector.

In this case, the alternative hypothesis has the form

$$H_1: \underline{p}_1 \neq \underline{p}_2 \neq \underline{p} \qquad (2)$$

for at least one category.

The criterion for testing these hypotheses at the α level is given by the following relation

$$\hat{X}^2_{n_1,n_2} = \frac{1}{\omega(1-\omega)} \left(\sum_{j=1}^{N} \omega_j v_{1j} - n_1 \omega \right),$$

where $\omega_j = \frac{v_{1j}}{(v_{1j}+v_{2j})}$, $\omega = \frac{n_1}{(n_1+n_2)}$.

For large values of n_1 and n_2, the chi-square homogeneity test rule reads: H_0 is rejected and $H_1 \leftrightarrow \hat{X}^2_{n_1,n_2} > \chi^2_{1-\alpha;\,N-1}$ is accepted, here $\chi^2_{1-\alpha;\,N-1}$ is the critical value of the chi-square test with the degree of freedom $N-1$.

Further, using the above reasoning, we proceed to the construction of the CC.

Results and discussions.

1). CC - λ.

Let the following hypotheses $H_0: F(x) = G(x)$ and $H_1: F(x) > G(x)$ be sequentially tested at certain times $t = 1,2, \dots, l$.

According to the alternative hypothesis H_1, we see that we need to build a control chart with an upper control boundary: UCL_λ.

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For the control value we take the following value $\lambda_t = \max_{-\infty<x<\infty}(G_{mt}(x) - F_{nt}(x))$ here $= 1,2, \dots, l$ sampling moments. We define UCL_λ using the following theorem.

Theorem 1. Let $\alpha = 0{,}01$ and the hypothesis H_0 is true. Then the following formula holds: $UCL_\lambda = 1.63\sqrt{\frac{n+m}{n \cdot m}}$.

Corollary (Implementation CC-λ). Let the definition of time points $t = 1,2, \dots, l$ $\lambda_t > UCL_\lambda$, then the alternative hypothesis is true $H_1: F(x) > G(x)$.

When implementing the CC-λ on the horizontal axis, we note the moments of sampling and on the y-axis of the value λ_t. Through the point UCL_λ on the y-axis we draw a straight line parallel to the horizontal axis. Next, mark the points on the plane with coordinates $(t;\ \lambda_t)$ and join them with segments. The resulting diagram determines the course of the pedagogical experiment.

From the corollary, we have that if $\lambda_t > UCL_\lambda$ the experiment gives a positive result at time t, and otherwise a negative result and, therefore, it is necessary to intervene in the process to clarify the plans for the experiment.

To implement the CC, you need to conduct an experiment. Here we show the results of three experiments to explain the meaning of solving the problem.

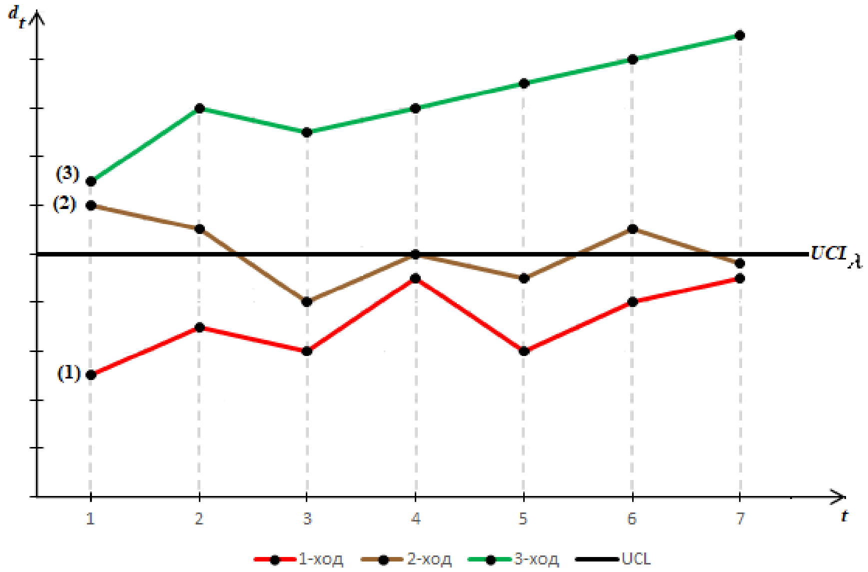

Fig.1. Homogeneity check with CC – λ.

(1) - the course of the first experiment (red line).

(2) - the course of the second experiment (yellow line).

(3) - the course of the third experiment (green line).

Figure 1 shows that in (1) and (2) cases, the experiment showed an unsatisfactory result. And in case (3), the differences between the two distributions are significant.

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2). Chi-square control chart (CC-χ^2).

Here we construct this map in the case of $N = 4$. At the same time, X and Y have four categories: 5 ($j = 1$), 4 ($j = 2$), 3 ($j = 3$) and 2 ($j = 4$) as assessments of the studied objects. Then hypotheses (1) and (2) take the following form

$$H_0: p_{11} = p_{21} = p_1;\ p_{12} = p_{22} = p_2;\ p_{13} = p_{23} = p_4;\ p_{14} = p_{24} = p_4$$

$$H_1: p_{11} \neq p_{21} \neq p_1;\ p_{12} \neq p_{22} \neq p_2;\ p_{13} \neq p_{23} \neq p_4;\ p_{14} \neq p_{24} \neq p_4$$

for at least one category.

Wherein

$$\hat{X}^2_{n_1,n_2} = \frac{1}{\omega(1-\omega)}\left(\sum_{j=1}^{N} \omega_j v_{1j} - n_1\omega\right).$$

The upper control limit of the desired CC is found from the fact that for large values of n_1 and n_2, the following relation holds:

$\hat{X}^2_{n_1,n_2} \to \chi^2_{1-\alpha;\,3}$.

Theorem 2. Let $\alpha = 0{,}01$ and the hypothesis H_0 is true, then the following formula holds for finding UCL_{χ^2}

$$UCL_{\chi^2} = n_1\omega + 0{,}115 \cdot \omega(1-\omega).$$

Consequence. If $g(\underline{X}, \underline{Y}) > UCL_{\chi^2}$, then the alternative hypothesis to H_0 is true.

The proofs of Theorems 1 and 2 are given by constructing a 99% confidence interval for control values. Here we prove Theorem 2.

From the limit relation (3) we have

$P\left(\hat{X}^2_{n_1,n_2} < \chi^2_{0{,}99;\,3}\right) = 0{,}99.$

This is equivalent to the relation

$$P\left(\sum_{j=1}^{4} \omega_j v_{1j} < n\omega + \omega(1-\omega)\chi^2_{0{,}99;3}\right) = 0{,}99.$$

Further, from the rule for choosing the control value and UCL_{χ^2} we have

$$g(\underline{X}, \underline{Y}) = \sum_{j=1}^{4} \omega_j v_{1j}$$

$UCL_{\chi^2} = n_1\omega + 0{,}115 \cdot \omega(1-\omega).$

The theorem has been proven.

To illustrate Theorem 2, we will consider an experiment aimed at identifying four properties of learning English RWSL (Reading; Writing; Speaking; Listening) in the old and new textbook for technical specialties. Forming a control and experimental group of students who studied according to the old and new books,

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samples were collected reflecting the knowledge of students and, according to it, students were divided into four categories. Testing students' knowledge in the experiment was carried out four times separately on the properties of RWSL. The constructed CC-χ^2on the basis of Theorem 2 and the corollary looks as follows.

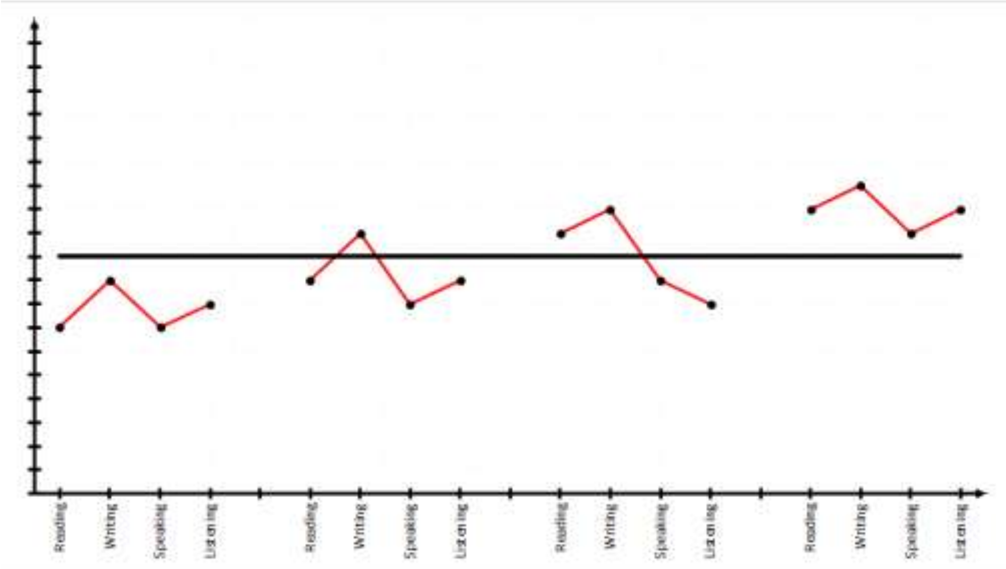

Fig.2. Homogeneity Check with CC - χ^2.

From the Control Chart diagram, we see that the experimenter had to work hard on the two properties S and L until the complete restoration of all RWSL properties.

In the end, we note some additional functions of the Control Chart:

• A control chart is a convenient method for monitoring pedagogical experiments. They are used right on the spot where instability testing is required and determines simple (random) and special reasons that bring the process into an uncontrollable state;

• CC help to improve the quality of the learning process;

• If on the basis of diagrams, the possible causes of instability are made, then using them the pedagogical process can be brought to a stable state.

The Control Chart method is universal in the sense that statistical inferences are given in diagrams. In addition, these diagrams are saved as a document of the experiments performed, and analysis of the results of several such experiments can provide predictive patterns.

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