

TRIGONOMETRIK WEIGHT OPTIMAL KVADRATUR OF THE FORMULA OF THE ERROR FUNCTIONAL OF NORM

<https://doi.org/10.5281/zenodo.10009917>

Baxromjon Bozarov Ilxomovich

Muhammad al-Khwarizmi 's Fergana branch of the tashkent university of information technologies.

b.bozarov@mail.ru

1. ACCESS

As it is known, the following

$$\int_0^1 p(x)\varphi(x)dx \cong \sum_{\beta=1}^N C[\beta]\varphi[\beta] \rightarrow \rightarrow \rightarrow \rightarrow (1)\P$$

kvadratur formula $p(x)$ weight kvadratur as the formula is referred to [1].

The above (1) kvadratur the formula to this

$$\ell(x) = \varepsilon_{[0,1]}(x)p(x) - \sum_{\beta=0}^N C[\beta]\delta(x - [\beta]) \rightarrow \rightarrow \rightarrow (2)\P$$

wholesale of view the error functional of suitable comes up. Thisnda $\varepsilon_{[0,1]}(x)$ this [0,1] of the incision xarakteristik function, $\delta(x)$ those who received the delta - function [1].

Such a weight kvadratur of the formula construction with h. m. Shadimetov, a. r. Hayotov , and others too many scienceiy surveys take are go [1,2].

The above $p(x)$ weight (1) kvadratur the formula of error

$$\ell(\varphi) = (\ell, \varphi) = \int_{-\infty}^{+\infty} \ell(x)\varphi(x)dx = \int_0^1 p(x)\varphi(x)dx - \sum_{\beta=0}^N C[\beta]\varphi[\beta] \rightarrow (3)\P$$

as is. ¶

Given (1) kvadratur to the formula corresponding kelivchi (2) the error functional with the above (3) addition is the error you get can. This with together (3) the error for the $|(\ell, \varphi)|$ option, the value of minimizing the issue important it is. This type of issues with [] to work in scientific research to bring you this was going.

That is, Kosh - Schwartz inequality from (1) kvadratur the formula for the absolute value φ of the function norm and (2) the error functional of norm multiples with from the top is charged [1-10]

$$|(\ell, \varphi)| \leq \|\varphi\|_{L_2^{(m)}} \cdot \|\ell\|_{L_2^{(m)*}}$$

This $L_2^{(m)*}$ space $L_2^{(m)}$ - to-space methodology'shma space. Xin the series to find the general appearance of the functional norms granted thegini bill s. l. Sobolev [1-10]. Worth mentioning, $p(x)$ the weight (1) kvadratur corresponding to formula (2) the norm of the error functional of the general kvadrati ko'to be rinib h. m. sh isdetected by adimetov [1.2]. Accordingly, the kamong (2) of the error functional of norm kvadrati

$$\|\ell\|_{L_2^{(m)*}}^2 = (-1)^m \left(\sum_{\beta=0}^N \sum_{\gamma=0}^N C[\beta]C[\gamma] \frac{([\beta]-[\gamma])^{2m-1}}{2 \cdot (2m-1)!} - 2 \sum_{\beta=0}^N C[\beta] \int_0^1 p(x) \frac{|x-[\beta]|^{2m-1}}{2 \cdot (2m-1)!} dx + \int_0^1 \int_0^1 p(x)p(y) \frac{|x-y|^{2m-1}}{2 \cdot (2m-1)!} dx dy \right)$$

ko'in rinib.

Now we $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ the weight kvadratur formlarning the error functional of norm of general appearance, finding the issue of looking at we are out.

2. THE NORM OF THE ERROR FUNCTIONAL OF THE FORMULA OF BEATS TRIGONOMETRIK KVADRATUR

This

$$\int_0^1 \sin(2\pi\omega x)\varphi(x)dx \cong \sum_{\beta=0}^N C_s[\beta]\varphi[\beta] \quad (4)$$

and

$$\int_0^1 \cos(2\pi\omega x)\varphi(x)dx \cong \sum_{\beta=0}^N C_c[\beta]\varphi[\beta] \quad (5)$$

done in the form of $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ the weight kvadratur the formula depending on the out ofyli we have. This is given in (4) and (5) trigonometrik beats kvadratur formula fit in, $C_s[\beta]$ and $C_c[\beta]$ the world kvadratur the formula of the coefficient, $[\beta]=h\beta$, $h = \frac{1}{N}$, N - natural number [at 3.4]. At the same time (1) and (2) trigonometrik weightkvadratur li formualalarga of functional fits the following error

$$\ell_s(x) = \varepsilon_{[0,1]}(x) \sin(2\pi\omega x) - \sum_{\beta=0}^N C_s[\beta]\delta(x - [\beta]) \quad (6)$$

and

$$\ell_c(x) = \varepsilon_{[0,1]}(x) \cos(2\pi\omega x) - \sum_{\beta=0}^N C_c[\beta]\delta(x - [\beta]). \quad (7)$$

The above (4) and (5) respectively trigonometrik beats kvadratur formula for the error as follows

$$(\ell_s, \varphi) = \int_0^1 \sin(2\pi\omega x)\varphi(x)dx - \sum_{\beta=0}^N C_s[\beta]\varphi[\beta]$$

and

$$(\ell_c, \varphi) = \int_0^1 \cos(2\pi\omega x)\varphi(x)dx - \sum_{\beta=0}^N C_c[\beta]\varphi[\beta]$$

determined.

That is in addition to, $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ a weight kvadratur formulasto the corresponding which has ℓ_s and ℓ_c error funksionallari the following terms and conditions too fulfill the requirements are

$$(\ell_s, x^\alpha) = 0 \quad \text{va} \quad (\ell_c, x^\alpha) = 0, \quad \alpha = 0, 1, \dots, m-1.$$

Above ko'browsing it should be noted that $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ the weight kvadratur the formula $(m-1)$ - level ko'ph clear.

To us it is known as Kosh - Shthe vart from disparity $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ weight with optimal kvadratur formulas's the error absolute value φ funkof siya norms and ℓ_s and ℓ_c the error funksionalarei norms ko'paytmasi with from the top is charged

$$|(\ell_s, \varphi)| \leq \|\varphi\|_{L_2^{(m)}} \cdot \|\ell_s\|_{L_2^{(m)*}}$$

and

$$|(\ell_c, \varphi)| \leq \|\varphi\|_{L_2^{(m)}} \cdot \|\ell_c\|_{L_2^{(m)*}}$$

Therefore, $L_2^{(m)}(0,1)$ in space $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ weight kvadratur formulasto the corresponding has, $L_2^{(m)*}(0,1)$ the joint space corresponding ℓ_s and ℓ_c the error funkin zionwith the world norms kvadrati follows determinegan [4.5]

[4.5]

$$\begin{aligned} \|\ell_\varepsilon\|_{L_2^{(m)*}}^2 &= (-1)^m \left(\sum_{\beta=0}^N \sum_{\gamma=0}^N C_\varepsilon[\beta] C_\varepsilon[\gamma] \frac{|\beta - \gamma|^{2m-1}}{2 \cdot (2m-1)!} \right. \\ &- 2 \sum_{\beta=0}^N C_\varepsilon[\beta] \int_0^1 \sin(2\pi\omega x) \frac{|x - \beta|^{2m-1}}{2 \cdot (2m-1)!} dx \\ &\left. + \int_0^1 \int_0^1 \sin(2\pi\omega x) \sin(2\pi\omega y) \frac{|x - y|^{2m-1}}{2 \cdot (2m-1)!} dx dy \right) \end{aligned} \quad (8)$$

and

$$\begin{aligned} \|\ell_\varepsilon\|_{L_2^{(m)*}}^2 &= (-1)^m \left(\sum_{\beta=0}^N \sum_{\gamma=0}^N C_\varepsilon[\beta] C_\varepsilon[\gamma] \frac{|\beta - \gamma|^{2m-1}}{2 \cdot (2m-1)!} \right. \\ &- 2 \sum_{\beta=0}^N C_\varepsilon[\beta] \int_0^1 \cos(2\pi\omega x) \frac{|x - \beta|^{2m-1}}{2 \cdot (2m-1)!} dx \\ &\left. + \int_0^1 \int_0^1 \cos(2\pi\omega x) \cos(2\pi\omega y) \frac{|x - y|^{2m-1}}{2 \cdot (2m-1)!} dx dy \right). \end{aligned} \quad (9)$$

TRIGONOMETRIK THE OPTIMAL FORMULA WEIGHT KVADRATUR $m = 1, 2$ THE CASES FOR THE NORMS OF THE ERROR FUNCTIONAL

Do this to $L_2^{(m)}(0,1)$ space $\sin(2\pi\omega x)$ and $\cos(2\pi\omega x)$ weight is optimal kvadratur formulas for $m = 1, 2$ the cases for [4,5] is given in *Teorema 3* and *Teorema 5* s's of the following results come in and stir.

Results 1. *Sobolevning $L_2^{(1)}(0,1)$ in space $\sin(2\pi\omega x)$ beats optimal kvadratur 's the formula, $\omega \in \square$ and $\omega \notin \square$ toe'at present, in the sense sar coefficient for the following equal as well as proper*

$$C_s[0] = h \left[\frac{1}{2\pi\omega h} - \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^2 \frac{\cos(\pi\omega h)}{2\sin(\pi\omega h)} \right],$$

$$C_s[\beta] = h \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^2 \sin(2\pi\omega[\beta]), \quad \beta = 1, 2, \dots, N-1,$$

$$C_s[N] = h \left[-\frac{\cos(2\pi\omega)}{2\pi\omega h} + \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^2 \frac{\cos(2\pi\omega - \pi\omega h)}{2\sin(\pi\omega h)} \right].$$

here $[\beta] = h\beta$, $h = \frac{1}{N}$.

The results 2. Sobolevning $L_2^{(2)}(0,1)$ in space $\sin(2\pi\omega x)$ beats optimal kvadratur 's the formula, $\omega \in \square$ and $\omega \notin \square$ toe'at present, in the sense sar coefficient for the following as well as equal o'rinli

$$C_s[0] = h \left[\frac{1}{2\pi\omega h} - \frac{K_{2,\omega} \cos(\pi\omega h)}{2\sin(\pi\omega h)} + \frac{m_{s,1}q_1 - n_{s,1}q_1^N}{q_1 - 1} \right],$$

$$C_s[\beta] = h \left[K_{2,\omega} \sin(2\pi\omega[\beta]) + m_{s,1}q_1^\beta + n_{s,1}q_1^{N-\beta} \right], \quad \beta = 1, 2, \dots, N-1,$$

$$C_s[N] = h \left[-\frac{\cos(2\pi\omega)}{2\pi\omega h} + \frac{K_{2,\omega} \cos(2\pi\omega - \pi\omega h)}{2\sin(\pi\omega h)} + \frac{-m_{s,1}q_1^N + n_{s,1}q_1}{q_1 - 1} \right]$$

here $[\beta] = h\beta$, $h = \frac{1}{N}$ and $q_1 = \sqrt{3} - 2$ the second level Eyley – Frobenius ko'ph

$E_2(x)$'s root,

$$K_{2,\omega} = \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^4 \frac{3}{2 + \cos(2\pi\omega h)},$$

$m_{s,1}$ and $n_{s,1}$ s following linear tenglmalar from the system is determined

$$m_{s,1} + n_{s,1}q_1^N = 0, |$$

The results 3. Sobolevning $L_2^{(1)}(0,1)$ in space $\cos(2\pi\omega x)$ beats optimal kvadratur 's the formula, $\omega \in \square$ and $\omega \notin \square$ toe'at present, in the sense sar coefficient for the following as well as equal o'rinli

$$C_c[0] = \frac{h}{2} \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^2,$$

$$C_c[\beta] = h \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^2 \cos(2\pi\omega[\beta]), \quad \beta = 1, 2, \dots, N-1,$$

$$C_c[N] = h \left(\frac{\sin(2\pi\omega)}{2\pi\omega h} - \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^2 \frac{\sin(2\pi\omega - \pi\omega h)}{2\sin(\pi\omega h)} \right),$$

here $[\beta] = h\beta$, $h = \frac{1}{N}$.

The results 4. Sobolevning $L_2^{(2)}(0,1)$ in space $\cos(2\pi\omega x)$ beats optimal kvadratur 's the formula, $\omega \in \square$ and $\omega \notin \square$ toe'at present, in the sense sar coefficient for the following as well as equal o'rinli

$$C_c[0] = h \left[\frac{K_{2,\omega}}{2} + \frac{m_{e,1}q_1 - n_{e,1}q_1^N}{q_1 - 1} \right],$$

$$C_c[\beta] = h \left[K_{2,\omega} \cos(2\pi\omega[\beta]) + m_{e,1}q_1^\beta + n_{e,1}q_1^{N-\beta} \right], \quad \beta = 1, 2, \dots, N-1,$$

$$C_c[N] = h \left[\frac{\sin(2\pi\omega)}{2\pi\omega h} - \frac{K_{2,\omega} \sin(2\pi\omega - \pi\omega h)}{2\sin(\pi\omega h)} + \frac{-m_{e,1}q_1^N + n_{e,1}q_1}{q_1 - 1} \right]$$

here $[\beta] = h\beta$, $h = \frac{1}{N}$ and $q_1 = \sqrt{3} - 2$ the second level Eyler – Frobenius

ko'ph $E_2(x)$'s root,

$$K_{2,\omega} = \left(\frac{\sin(\pi\omega h)}{\pi\omega h} \right)^4 \frac{3}{2 + \cos(2\pi\omega h)},$$

$m_{e,1}$ and $n_{e,1}$ the world the following linear equations from the system is determined

$$m_{e,1} + n_{e,1}q_1^N = -\frac{(1-q_1)^2}{q_1} \left[\frac{1}{(2\pi\omega h)^2} + \frac{K_{2,\omega}}{2(\cos(2\pi\omega h) - 1)} \right],$$

$$m_{e,1}q_1^N + n_{e,1} = -\frac{(1-q_1)^2 \cos(2\pi\omega)}{q_1} \left[\frac{1}{(2\pi\omega h)^2} + \frac{K_{2,\omega}}{2(\cos(2\pi\omega h) - 1)} \right].$$

Given above are the results 1 – 4 results are respectively from (8) and (9) of the norm of the error functional kvadrati for $m=1$ in the case of after some calculations, you can bring the following confirmation.

Teorema 1. $L_2^{(1)}$ in space (4) optimal kvadratur of the formula (8) the error functional of norm kvadrati for the following check seats

$$\|e_\varepsilon\|^2 = \frac{\sin(4\pi\omega)}{4(2\pi\omega)^3} + \frac{1}{2(2\pi\omega)^2} - \frac{1 - \cos(2\pi\omega h)}{(2\pi\omega)^4 h^2} - \frac{\sin(\pi\omega h) \sin(4\pi\omega)}{2h(2\pi\omega)^4 \cos(\pi\omega h)}.$$

Teorema 2. $L_2^{(1)}$ in space (5) optimal kvadratur formula (9) of the error functional of norm kvadrati for the following check seats

$$\|e_\varepsilon\|^2 = \frac{1}{2(2\pi\omega)^2} - \frac{\sin(2\pi\omega) \cos(2\pi\omega)}{2(2\pi\omega)^3} - \frac{1 - \cos(2\pi\omega h)}{(2\pi\omega)^4 h} \left(\frac{1}{h} - \frac{\sin(2\pi\omega) \cos(2\pi\omega)}{\sin(2\pi\omega h)} \right).$$

Found the error functional of norming to approach the procedure follows

$$O((N+|\omega|)^{-1}).$$

SUMMARY

Trigonometrik in sobolev space beats kvadratur formula determined optimal koeffisiyentlar using the error functional of norm isnd. This built - optimal kvadratur formula using geomatematika, waves theory, restore images, elektrodinamika and computer tomography , which occurs practical issues in solving applied.

LITERATURE RECENT'YXATI:

1. Шадиметов Х.М. Оптимальные решетчатые квадратурные и кубатурные формулы в пространствах Соболева. – Ташкент: Фан ва технология, 2019.
2. Shadimetov Kh.M., Hayotov A.R., Bozarov B.I. Optimal quadrature formulas for oscillatory integrals in the Sobolev space. Journal of inequalities and applications. Springer. 2022.
3. Hayotov A.R., Bozarov B.I. Optimal quadrature formula with cosine weight function. Problems of Computational and Applied Mathematics. 2021. Vol. 4, No 34, pp 106-118.

4. Bozarov, B. I. An optimal quadrature formula with $\sin x$ weight function in the Sobolev space. Uzbekistan academy of sciences vi romanovski institute of mathematics, 2019, pp 47.

5. Bozarov B.I., Shaev A.K. Norm of the error functional for the optimal quadrature formula with cosine weight in the Sobolev space. Problems of Computational and Applied Mathematics. 2023, Vol 50, No:3 (1), pp.

6. Далиев, Б. С. (2021). Оптимальный алгоритм решения линейных обобщенных интегральных уравнений Абеля. Проблемы вычислительной и прикладной математики, 5(35), 120-129.

7. Shadimetov, K. M., & Daliev, B. S. (2022). Optimal formulas for the approximate-analytical solution of the general Abel integral equation in the Sobolev space. Results in Applied Mathematics, 15, 100276.

8. Далиев, Б. С. (2022). О Численном Решении Линейных Обобщенных Интегральных Уравнений Абеля. Periodica Journal of Modern Philosophy, Social Sciences and Humanities, 13, 191-198.

9. Rashidjon, R., & Sattorov, A. (2021). Optimal Quadrature Formulas with Derivatives in the Space. Middle European Scientific Bulletin, 18, 233-241.

10. Расулов, Р., Сатторов, А., & Махкамова, Д. (2022). Вычисление Квадрат Нормы Функционала Погрешности Улучшенных Квадратурных Формул В Пространстве. Central asian journal of mathematical theory and computer sciences, 3(4), 114-122.

11. Tukhtasinov, D. (2018). Development of logical thinking of pupils of 5-9th grades in the lessons of mathematics. Zbiór artykułów naukowych recenzowanych, 209(22), 586-587.

12. Farhodovich, T. X. D. (2023). Boshlang 'ich sinf o 'quvchilarining tafakkurini rivojlantirishning psixologik va pedagogik jihatlari. Ijtimoiy fanlarda innovasiya onlayn ilmiy jurnali, 3(3), 24-28.

13. To'xtasinov, D. F., Gafurova, M. A., & qizi Abdullayeva, S. H. (2023). BOSHLANG 'ICH SINFDIY IJODIY TAFAKKURNI RIVOJLANTIRISHNING PEDAGOGIK ASOSLARI. Educational Research in Universal Sciences, 2(4), 729-733.

14. To'xtasinov, D. F., & qizi Abdupattoyeva, D. A. (2023). BOSHLANG 'ICH SINFDIY O 'QUVCHILARINING GEOMETRIK TASAVVURINI SHAKLLANTIRISHDA FOYDALANILADIGAN METODLAR. Educational Research in Universal Sciences, 2(4), 771-773.

15. To'xtasinov, D. F., & qizi Abdullayeva, S. H. (2023). MATEMATIKA DARSLARIDA IJODIY TAFAKKURNI RIVOJLANTIRISH SHARTLARINING

DIDAKTIK KOMPLEKSINI AMALIYOTDA QO‘LLASH YO‘LLARI. Educational Research in Universal Sciences, 2(2), 613-616.