International Journal of Education, Social Science & Humanities.
Finland Academic Research Science Publishers
ISSN: 2945-4492 (online) | (SJIF) = 7.502 Impact factor
Volume-11| Issue-12| 2023 Published: |22-12-2023|

BOSHQARUVLARI GEOMETRIK CHEGARALANISHLI HOL UCHUN QUVISH-QOCHISH MASALASI

https://doi.org/10.5281/zenodo.10442653

Xolmirzayeva Guljaxon Ibrohimjon qizi
ADU, doktoranti. xolmirzayeva1991@mail.ru

Faraz qilaylik, R^n fazoda qarama-qarshi maqsadli ikkita o'yinchilar, ya'ni P o'yinchi (quvlovchi) va E o'yinchi (qochuvchi) harakatlanayotgan bo'lib, P o'yinchi biror yo'nalishda E o'yinchini ta'qib qilsin. Quvlovchi va qochuvchilarning fazodagi holatlarini mos ravishda x va y orqali belgilaymiz. Ularning harakatlari quyidagi differensial tenglamalar va boshlang'ich shartlar orqali berilgan bo'lsin:

$$\text{P}: \ \dot{x} = u, \ \ x(0) = x_0, \qquad (1)$$

$$E: \ \dot{y} = v, \ \ y(0) = y_0, \qquad (2)$$

bu yerda $x, y, u, v, x_0, y_0 \in R^n$, $n \geq 2$, x_0, y_0 nuqtalar mos ravishda quvlovchi va qochuvchining $t = 0$ vaqtdagi boshlang'ich vaziyatlari bo'lib, $x_0 \neq y_0$ deb qaraymiz. Shuningdek, u va v parametrlar obyektlarning boshqaruv parametrik hisoblanadi.

Quvlovchining u boshqaruv parametri $u(\cdot):[0;+\infty) \to R^n$ akslantirishni bajaruvchi t bo'yicha o'lchovli funksiya sifatida tanlanadi va quyidagi chegaralanishni qanoatlantirishini talab etamiz :

$$|u(t)| \leq \rho_0 a^{-kt} + a^{kt}, \text{ deyarli barcha } t \geq 0, \qquad (3)$$

bu yerda $0 < a < 1,\ a > 1,\ k > 0,\ \rho_0 > 0.$

Qochuvchining v boshqaruv parametri $v(\cdot):[0;+\infty) \to R^n$ akslantirishni bajaruvchi t bo'yicha o'lchovli funksiya sifatida tanlanadi va quyidagi chegaralanishni qanoatlantirishini talab etamiz :

$$|v(t)| \leq \sigma_0 a^{-kt} + a^{kt}, \text{ deyarli barcha } t \geq 0, \qquad (4)$$

bu yerda $0 < a < 1,\ a > 1,\ k > 0,\ \sigma_0 > 0.$

(1) tenglamaning u parametr P quvlovchining (3) chegaralanishni qanoatlantiruvchi $u(\cdot)$ o'lchovli funksiyalar sinfini $U_{\rho_0}^{a,k}$ bilan belgilaymiz.

Xuddi shu kabi, E qochuvchining (4) chegaralanishni qanoatlantiruvchi $v(\cdot)$ o'lchovli funksiya sinfini $V_{\sigma_0}^{a,k}$ bo'lib belgilanadi.

International Journal of Education, Social Science & Humanities.
Finland Academic Research Science Publishers
ISSN: 2945-4492 (online) | (SJIF) = 7.502 Impact factor
Volume-11| Issue-12| 2023 Published: |22-12-2023|

Quyidagicha belgilash kiritaylik:

$\varphi(t)=\rho_0 a^{-kt}+a^{kt}$,

$\psi(t)=\sigma_0 a^{-kt}+a^{kt}$.

Ta'rif-1. $\forall u(\cdot)\in U_{\rho_0}^{a,k}$ va $\forall v(\cdot)\in V_{\sigma_0}^{a,k}$ boshqaruv funksiyalar uchun (1) va (2) tenglamalardan hosil bo'lgan quyidagi ifodalar mos ravishda quvlovchi va qochuvchining harakat trayektoriyasi deyiladi:

$$x(t)=x_0+\int_0^t u(s)ds, \qquad (5)$$

$$y(t)=y_0+\int_0^t v(s)ds. \qquad (6)$$

Ta'rif-2. $u(t,v)$ funksiya t quvlovchining strategiyasi deb aytiladi, agar

1) $u(t,v)$ funksiya har bir fiksirlangan v uchun t bo'yicha Lebeg o'lchovli bo'lsa;

2) $u(t,v)$ funksiya har bir fiksirlangan t uchun v bo'yicha Borel o'lchovli bo'lsa;

3) $\forall v(\cdot)\in V_{\sigma_0}^{a,k}$ boshqaruv funksiya uchun $u(t,v(t))\in U_{\rho_0}^{a,k}$ munosabat o'rinli bo'lsa, bu yerda $u(t,v(t))$ funksiya $u(t,v)$ strategiyalar realizatsiyasi (qo'llanishi) deyiladi.

Ta'rif-3. $u(t,v)$ strategiya yutuqli deyiladi agar, $\forall v(\cdot)\in V_{\sigma_0}^{a,k}$ boshqaruv funksiya uchun

$\dot{x}=u(t,v(t)),\ x(0)=x_0,$ (7)

$\dot{y}=v(t),\ y(0)=y_0$ (8)

Koshi masalasining $x(t)$ va $y(t)$ yechimlari uchun $\exists\theta\in[0,T]$ vaqt topilsaki, bu vaqtda $x(\theta)=y(\theta)$ tenglik hosil bo'lsa, bu yerda T son kafolatlangan yutuqli vaqt deyiladi.

Ta'rif-4. $v^*(t):R_+\to V_{\sigma_0}^{a,k}$ funksiya E qochuvchining strategiyasi deyiladi, agar $v(t)$ funksiya t bo'yicha Lebeg o'lchovli funksiya bo'lsa.

Ta'rif-5. $v=v^*(t)$ strategiya yutuqli deyiladi, agar $\forall u(\cdot)\in U_{\rho_0}^{a,k}$ boshqaruv funksiya uchun

$\dot{x}=u(t),\ \ x(0)=x_0,$ (9)

$\dot{y}=v^*(t),\ \ y(0)=y_0$ (10)

Koshi masalasi yechimlari $x(t)$ va $y(t)$ uchun $\forall t \geq 0$ qiymatlarda $x(t) \neq y(t)$ muonosabat bajarilsa.

Ta'rif-6. (1)-(4) quvish differensial o'yinida ushbu

$$u(t,v) = v = \lambda(t,v)\xi_0 \qquad (11)$$

funksiyaga P quvlovchining parallel quvish strategiyasi (Π-strategiya) deb ataymiz, bu yerda

$$\lambda(t,v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \varphi^2(t) - |v|^2}, \ \xi_0 = \frac{z_0}{|z_0|}, \ z_0 = x_0 - y_0.$$

Quyida $\lambda(t,v)$ funksiyaning aniqlanish shartini topamiz:

$$(\varphi(t) - |v|)(\varphi(t) + |v|) \geq 0,$$

$$\varphi(t) \geq |v| \ \Leftrightarrow \ \varphi(t) \geq \psi(t),$$

$$\rho_0 a^{-kt} + a^{kt} \geq \sigma_0 a^{-kt} + a^{kt},$$

$$\rho_0 \geq \sigma_0.$$

Teorema-1. Agar quyidagi shartlardan biri o'rinli bo'lsa, ya'ni

a) $0 < a < 1, \ \rho_0 > \sigma_0;$

b) $a > 1, \ \rho_0 > \sigma_0 + k|z_0|\ln a$

u holda (11) strategiya $[0, T_*]$ vaqt oralig'ida yutuqli bo'ladi, bu yerda

$$T_* = \frac{1}{k}\log_a\left(\frac{\rho_0 - \sigma_0}{\rho_0 - \sigma_0 - k|z_0|\ln a}\right).$$

Isbot. Faraz qilamiz, E qochuvchi biror $v(\cdot) \in V_{\sigma_0}^{a,k}$ boshqaruv funksiya tanlasin. U holda P quvlovchi shu funksiyaga mos $u(t, v(t))$ strategiyani qo'llaydi.

Agar $z(t) = x(t) - y(t)$ belgilash kiritsak, u holda (7)-(8) Koshi masalasidan yagona Koshi masalasiga kelamiz.

Ta'rif-7. (1)–(4) qochish differensial o'ynida ushbu

$$v^*(t) = -\psi(t)\xi_0 \qquad (12)$$

funksiyaga qochuvchining strategiyasi deb ataladi, bu yerda

$$\xi_0 = \frac{z_0}{|z_0|}, \ z_0 = x_0 - y_0.$$

Teorema- 2. Agar quyidagi shartlardan biri bajarilsa

a) $0 < a < 1, \ \rho_0 \leq \sigma_0;$

b) $a > 1, \ \rho_0 \leq \sigma_0 + k|z_0|\ln a$

International Journal of Education, Social Science & Humanities.
Finland Academic Research Science Publishers
ISSN: 2945-4492 (online) | (SJIF) = 7.502 Impact factor
Volume-11| Issue-12| 2023 Published: |22-12-2023|

u holda (1)–(4) qochish differensial o'yinida (12) strategiya yutuqli bo'ladi.

Isbot. Faraz qilaylik, P quvlovchi biror $u(\cdot)\in U_{\rho_0}^{a,k}$ boshqaruv funksiya tanlasin. U holda E qochuvchi (12) strategiyadan foydalanib harakatlanadi. Bundan (9), (10) Koshi masalalarini quyidagi yagona masalaga keltiramiz:

$$\begin{cases} \dot{z} = u(t) - v^*(t), \\ z(0) = z_0. \end{cases}$$

Endi hosil bo'lgan tenglamani integrallab, quyidagi yechimni hosil qilamiz:

$$z(t) = z_0 + \int_0^t u(s)ds - \int_0^t v^*(s)ds.$$

Hosil bo'lgan yechim funksiyani modul xossalaridan foydalanib, quyidan baholaymiz:

$$|z(t)| = \left| z_0 + \int_0^t u(s)ds - \int_0^t v^*(s)ds \right| = \left| z_0 - \int_0^t v^*(s)ds - \int_0^t -u(s)ds \right| \geq$$

$$\geq \left| z_0 + \int_0^t \psi(s)ds\xi_0 \right| - \left| \int_0^t -u(s)ds \right| \geq |z_0| + \int_0^t \psi(s)ds - \int_0^t |u(s)|ds \geq$$

$$\geq |z_0| + \int_0^t \psi(s)ds - \int_0^t \varphi(s)ds = |z_0| + \int_0^t [\sigma_0 a^{-ks} + a^{ks} - \rho_0 a^{-ks} - a^{ks}]ds =$$

$$= |z_0| + (\sigma_0 - \rho_0)\int_0^t a^{-ks}ds = |z_0| + \frac{\rho_0 - \sigma_0}{k\ln a}(a^{-kt} - 1) = \Lambda(t)$$

yoki

$$|z(t)| > \Lambda(t)$$

tengsizlikka ega bo'lamiz.

Quyida $\Lambda(t)$ funksiyani teoremada berilgan shartlar asosida quyidagicha tahlil asosida ko'rinadiki, barcha $t \geq 0$ uchun $\Lambda(t) > 0$ munosabat o'rinli bo'ladi va $|z(t)| > \Lambda(t)$ tengsizlikka ko'ra esa $|z(t)| > 0$ munosabatga ega bo'lamiz. Teorema 2 isbot bo'ldi.

FOYDALANILGAN ADABIYOTLAR RO'YXATI

1. Isaacs R. Differential games. John Wiley and Sons, New York. 1965, 340 p.
2. Pontryagin L.S. Selected Works. MAKS Press, Moscow. 2014, 551 p. (In Russian)

3. Krasovsky N.N., Subbotin A.I. Game-Theoretical Control Problems. Springer, New York. 1988, 517 p.

4. Azamov A.A., Samatov B.T. The Π-Strategy: Analogies and Applications. The Fourth International Conference Game Theory and Management. St. Petersburg: Leningrad. Univ. June 28-30, 2010, pp. 33-47.

5. Satimov N.Yu. Methods for Solving the Pursuit Problem in the Theory of Differential Games. Izd-vo NUUz, Tashkent, 2003. (In Russian)

6. Ibragimov G.I. Differential multi-person game with integral constraints for controls of the players. Izv. Vuzov, Matematika, 2004, No. 4, pp. 48-52.

7. Samatov B.T., Horilov M.A., Akbarov A.Kh. Differential games with non-stationary geometric constraints on controls. Lobachevskii Journal of Mathematics. Pleiades Publishing, 2022, Vol. 43, No. 1, pp 237-248.

8. Samatov B.T., Soyibboev U.B. Differential game with a lifeline for the inertial movements of players. Ural Mathematical Journal, 2021, Vol. 7, No. 2, pp. 94-109.