Volume-11| Issue-1| 2023 Research Article

TIP O'ZGARISH CHIZIG'I SILLIQ BO'LMAGAN PARABOLIK-GIPERBOLIK TENGLAMA UCHUN INTEGRAL ULASH SHARTLI CHEGARAVIY MASALA

https://doi.org/10.5281/zenodo.7548455

Gaziyev Qobiljon Solijonovich
Farg'ona davlat universiteti
Matematik analiz va differensial tenglamalar kafedrasi dotsenti
e-mail ksgaziev1965@gmail.com
A'zamov Valiahror Halimjon o'g'li
Farg'ona davlat universiteti
Matematika (yo'nalishlar bo'yicha) yo'nalishi magistranti
e-mail valiahror@mail.ru

Abstract:. Tip o'zgarish chizig'i silliq bo'lmagan parabolik-giperbolik tenglama uchun integral ulash shartli chegaraviy masalaning bir qiymatli yechilishi isbotlangan. Masala yechimining yagonaligi energiya integrallari usulida, mavjudligi esa integral tenglamalar usulida isbotlangan.

Keywords: parabolik-giperbolik teglama; integral ulash sharti; energiya integrallari; Grin funksiyasi; Dalamber formulasi; integral tenglama

Received: 17-01-2023
Accepted: 18-01-2023
Published: 22-01-2023

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BOUNDARY PROBLEM WITH INTEGRAL GLUING CONDITION FOR PARABOLIC-HYPERBOLIC EQUATION WITH NON-SMOOTH LINE OF TYPE-CHANGING

Gaziyev Qobiljon Solijonovich
A'zamov Valiahror Halimjon o'g'li

Abstract:. A unique solvability of a boundary problem with integral gluing condition for parabolic-hyperbolic equation with non-smooth line of type-changing has been proved. The uniqueness of the solution to the problem is proved by energy integral's method, the existence by the method of integral equations..

Keywords: Parabolic-hyperbolic equation; integral gluing condition; energy integrals; Green's function; d'Alambert's formula; integral equations

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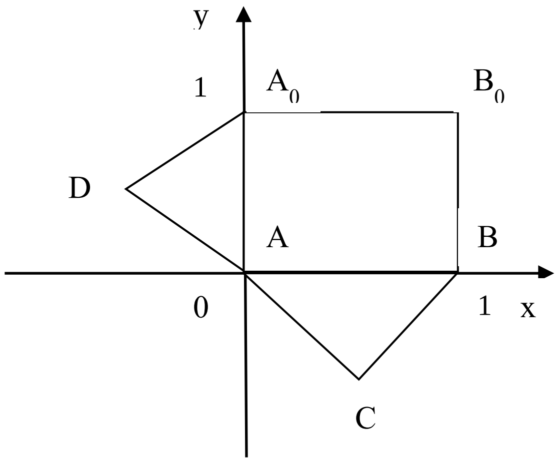
Bizga Ω sohada quyidagi tenglama berilgan bo'lsin:

$$\begin{cases} U_{xx} - U_y = 0, & (x,y) \in \Omega_0 \\ U_{xx} - U_{yy} = 0, & (x,y) \in \Omega_i, (i=1,2) \end{cases} \quad (1)$$

Ω soha esa quyidagicha $\Omega = \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup AB \cup AA_0$, bu yerda $A(0,0); A_0(0,1); B(1,0); B_0(1,1); C\left(\frac{1}{2}, -\frac{1}{2}\right); D\left(-\frac{1}{2}, \frac{1}{2}\right)$ ю



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Masala. (1) tenglamani qanoatlantiruvchi Ω sohada shunday $U(x,y)$ funksiya topilsinki, u $U(x,y)\in C^{2,1}_{x,y}(\Omega_0)\cap C(\bar{\Omega})\cap C^2(\Omega_i),(i=1,2)$ regulyarlik shartlarini hamda quyidagi shartlarni qanoatlantirsin:

$$U(x,y)\big|_{AC}=\psi_1(x),\qquad 0\le x\le\frac{1}{2},\qquad (2)$$

$$U(x,y)\big|_{AD}=\psi_2(y),\qquad -\frac{1}{2}\le x\le 0,\qquad (3)$$

$$U(x,y)\big|_{BB_0}=\varphi(y),\qquad 0\le y\le 1,\qquad (4)$$

$$U_y(x,+0)=I_1(U_y(x,-0)),\ U_y(x,+0)=U_y(x,-0),\quad 0<x<1,\qquad (5)$$

$$U_x(+0,y)=I_2(U_x(-0,y)),\ U_x(+0,y)=U_x(-0,y),\quad 0<y<1.\qquad (6)$$

Bu yerda I_1 va I_2 lar hozircha ixtiyoriy integral operatorlar.

Ω_0 sohada $U_{xx}-U_y=0$ tenglama uchun birinchi chegaraviy masala yechimini quyidagicha yozamiz [1]:

$$u(x,y)=\int_0^1\tau_1^+(\xi)G_1(x,y;\xi,0)d\xi+\int_0^y\tau_2^+(\eta)G_{1\xi}(x,y;0,\eta)d\eta-\int_0^y\varphi(\eta)G_{1\xi}(x,t;l,\eta)d\eta$$

bu yerda, $\tau_1^+(x)=U(x,0),\quad 0\le x\le 1,$

$\tau_2^+(y)=U(0,y),\quad 0\le y\le 1,\qquad \varphi(y)=U(1,y),\quad 0\le y\le 1.$

Ω_1 sohada $U_{xx} - U_{yy} = 0$ tenglamaning $U(x,-0) = \tau_1^-(x)$, $0 \le x \le 1$, $U_y(x,-0) = \nu_1^-(x)$, $0 < x < 1$boshlang'ich shartlarni qanoatlantiruvchi yechimini Dalamber formulasi orqali yozamiz [2]:

$$U(x,y) = \frac{1}{2}\left[\tau_1^-(x-y) + \tau_1^-(x+y)\right] + \frac{1}{2}\int_{x-y}^{x+y} \nu_1^-(z)dz .$$

Bu yechimni (2) shartga bo'ysundiramiz:

$$2\psi_1(x) = \tau_1^-(0) + \tau_1^-(2x) + \int_{2x}^{0} \nu_1^-(z)dz , \quad 0 \le x \le \frac{1}{2} .$$

Endi $2x$ ni x bilan almashtirsak

$$2\psi_1\left(\frac{x}{2}\right) = \tau_1^-(0) + \tau_1^-(x) - \int_0^x \nu_1^-(z)dz , \quad 0 \le x \le 1$$

hosil bo'ladi. Keyin bu tenglikni x bo'yicha differensiallasak

$$\psi_1'\left(\frac{x}{2}\right) = \tau_1'(x) - \nu_1^-(x) , \quad 0 < x < 1 \qquad (7)$$

Munosabatni olamiz.

Ω_2 sohada $U_{xx} - U_{yy} = 0$ tenglamaning $U(-0,y) = \tau_2^-(y)$, $0 \le y \le 1$, $U_x(-0,y) = \nu_2^-(y)$, $0 < y < 1$ boshlang'ich shartlarni qanoatlantiruvchi yechimini Dalamber formulasi orqali yozamiz [2]:

$$U(x,y) = \frac{1}{2}\left[\tau_2(y-x) + \tau_2(y+x)\right] - \frac{1}{2}\int_{y+x}^{y-x} \nu_2^-(z)dz .$$

Bu yechimni (3) shartga bo'ysundiramiz:

$$2\psi_2(y) = \tau_2^-(2y) + \tau_2^-(0) - \int_0^{2y} \nu_2^-(z)dz , \quad 0 \le y \le \frac{1}{2} .$$

Endi $2y$ ni y bilan almashtirsak,

$$2\psi_2\left(\frac{y}{2}\right) = \tau_2^-(y) + \tau_2^-(0) - \int_0^y \nu_2^-(z)dz , \quad 0 \le y \le 1$$

hosil bo'ladi. Keyin bu tenglikni y bo'yicha differensiallasak

$$\psi_2'\left(\frac{y}{2}\right) = \tau_2'(y) - \nu_2^-(y) , \quad 0 < y < 1 \qquad (8)$$

tenglikka ega bo'lamiz.

Masala yechimining yagonaligi.

Masala yechimi 2 ta u_1 va u_2 funksiyalar bo‘lsin. U holda $v = u_1 - u_2$ funksiyaga nisbatan quyidagi masalani qarashimiz mumkin bo‘ladi:

$$\begin{cases} v_{xx} - v_y = 0, & (x,y) \in \Omega_0 \\ v_{xx} - v_{yy} = 0, & (x,y) \in \Omega_i, (i=1,2) \end{cases}$$

$$v(x,y)\big|_{AC} = 0, v(x,y)\big|_{AD} = 0, \qquad v(x,y)\big|_{BB_0} = 0,$$

$$v_y(x,+0) = I_1(v_y(x,-0)), \ v_x(+0,y) = I_2(v_x(-0,y))$$

Bu esa bir jinsli tenglama uchun bir jinsli shartli masaladir. Agar ushbu masala yechimining $v(x,y) \equiv 0$ ekanligini ko‘rsata olsak, asosiy masalaning yechimi yagonaligi kelib chiqadi:

Dastlab, $v_{xx} - v_y = 0$ tenglamada $y \to +0$ holda limitga o‘tamiz va

$$\tau_1^{+\prime\prime}(x) - \nu_1^+(x) = 0$$

tenglamani olamiz. Bu yerda $\tau_1^+(x) = v(x,+0)$, $\nu_1^+(x) = v_y(x,+0)$ hamda ushbu tenglamani $\tau_1^+(x)$ ga ko‘paytirib (0,1) oraliqda integrallaymiz:

$$\int_0^1 \tau_1^+(x)\tau_1^{+\prime\prime}(x)dx - \int_0^1 \tau_1^+(x)\nu_1^+(x)dx = 0.$$

Ushbu integralni bo‘laklab integrallaymiz:

$$\tau_1^+(x)\tau_1^{+\prime}(x)\Big|_0^1 - \int_0^1 \left[\tau_1^{+\prime}(x)\right]^2 dx - \int_0^1 \tau_1^+(x)\nu_1^+(x)dx = 0.$$

$\tau_1^+(0) = \tau_1^+(1) = 0$ ekanligini inobatga olsak yuqoridagi ifoda quyidagi ko‘rinishga keladi:

$$\int_0^1 \left[\tau_1^{+\prime}(x)\right]^2 dx + \int_0^1 \tau_1^+(x)\nu_1^+(x) = 0$$

Bundan $\int_0^1 \left[\tau_1^{+\prime}(x)\right]^2 dx \ge 0$ bo‘lgani uchun $\int_0^1 \tau_1^+(x)\nu_1^+(x) \ge 0$ ekanligini ko‘rsata olsak yechim yagonaligini ko‘rsatishimiz mumkin bo‘ladi. Buning uchun I_1 operatorni quyidagicha tanlaymiz:

$$I_1\left(f(x)\right) = \int_0^x f(z)K(x,z)dz.$$

Demak,

$$J_1 = \int_0^1 \tau_1^+(x)\nu_1^+(x)dx = \int_0^1 \tau_1^+(x)I_1\left(\tau_1^+(x)\right)dx = \int_0^1 \tau_1^+(x)dx\int_0^x \tau_1^+(z)K(x,z)dz =$$

$$= \int_0^1 \tau_1^+(x)dx\left[\tau_1^+(z)K(x,z)\Big|_0^x - \int_0^x \tau_1^+(z)\frac{\partial}{\partial z}K(x,z)dz\right] =$$

$$= \int_0^1 \tau_1^2(x)K(x,x)dx - \int_0^1 \tau_1(x)dx\int_0^x \tau_1^+(z)\frac{\partial}{\partial z}K(x,z)dz.$$

Bundan $K(x,x) \geq 0$deb olib , $K(x,z)$ uchun quyidagi tenglikni o‘rinli deb olsak

$$\frac{\partial}{\partial z}K(x,z) = -K_1(x)K_1(z),$$

J_1 integralimizni quyidagicha ifodalashimiz mumkin bo‘ladi:

$$\int_0^1 \tau_1^2(x)K(x,x)dx + \frac{1}{2}\int_0^1 \frac{d}{dx}\left[\int_0^x \tau_1^+(z)K_1(z)\right]^2 dz =$$

$$= \int_0^1 \tau_1^2(x)K(x,x)dx + \frac{1}{2}\left(\int_0^1 \tau_1^+(z)K_1(z)dz\right)^2.$$

Shunday qilib yuqoridagi shartlarni qanoatlantiruvchi $K(x,z)$ funksiya uchun $\tau_1^-(x)$ funksiyani nolga tengligi kelib chiqadi. Bundan esa $\nu_1^-(x)$ funksiyani ham nolga tengligi kelib chiqadi.

Agar $\tau_1^-(x) = 0, \nu_1^-(x) = 0$ bo‘lsa Dalamber formulasidan $v(x,y)$ni $v(x,y) \equiv 0$ ekanligi kelib chiqadi, bu esa bizga yechim yagonaligini keltirib chiqaradi.

Endi $v_{xx} - v_y = 0$ tenglamani olib uni v ga ko‘paytirib Ω_0 soha bo‘yicha integrallaymiz:

$$\iint_{\Omega_0} v(v_{xx} - v_y)dxdy = 0 \text{ yoki}$$

$$\iint_{\Omega_0}\left[(vv_x)_x - v_x^{\,2} - \frac{1}{2}(v^2)_y\right]dxdy = 0.$$

Grin formulasini qo‘llab [2], soha bo‘yicha integralni soha chegarasi bo‘yicha integralga keltiramiz:

$$\int_{\partial\Omega_0} vv_x dy + \frac{1}{2}(v^2)dx - \iint_{\Omega_0} v_x^{\,2}dxdy = 0.$$

Endi chegara bo‘yicha integrallashni amalga oshirib olamiz:

$$\int_0^1 \frac{1}{2}\left[\tau_1^+(x)\right]^2 dx + \int_0^1 v(1,y)v_x(1,y)dy - \frac{1}{2}\int_0^1 v^2(x,1)dx - \int_0^1 \tau_2^+(y)v_2^+(y)dy -$$

$$-\iint_{\Omega_0} v_x^2(x,y)dxdy = 0.$$

$\tau_1^+(x)=0$, $v(1,y)=0$ ekanligidan integral quyidagi ko‘rinishga keladi:

$$\frac{1}{2}\int_0^1 v^2(x,1)dx + \int_0^1 \tau_2^+(y)v_2^+(y)dy + \iint_{\Omega_0} v_x^2(x,y)dxdy = 0\,.$$

Agar $\int_0^1 \tau_2^+(y)v_2^+(y)dy \ge 0$ ni ekanligini ko‘rsata olsak yechim yagonaligini ko‘rsatishimiz mumkin.

$$J_2 = \int_0^1 \tau_2^+(y)v_2^+(y)dy = \int_0^1 \tau_2^+(y)I_2(\tau_2{}'(y)dy = \left\{I_2(f(x)) = \int_0^x f(x)P(x,z)dz\right\} =$$

$$= \int_0^1 \tau_2^+(y)\left[\int_0^y \tau_2{}'(z)P(y,z)dz\right]dy = \int_0^1 \tau_2^+(y)\left[\tau_2^+(z)P(y,z)\Big|_0^y - \int_0^y \tau_2(z)\frac{\partial}{\partial z}P(y,z)dz\right]dy =$$

$$= \int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy - \int_0^1 \tau_2^+(y)\left[\int_0^y \tau_2(z)\frac{\partial}{\partial z}P(y,z)dz\right]dy =$$

$$= \left\{\frac{\partial}{\partial z}P(y,z) = -P_1(y)P_1(z)\right\} = \int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy +$$

$$+\int_0^1 \tau_2^+(y)\left[\int_0^y \tau_2(z)P_1(y)P_1(z)dz\right]dy = \int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy +$$

$$+\frac{1}{2}\int_0^1 \frac{d}{dy}\left[\int_0^y \tau_2^+(z)P_1(z)dz\right]^2 dy = \int_0^1 \left(\tau_2^+(y)\right)^2 P(y,y)dy + \frac{1}{2}\left[\int_0^1 \tau_2^+(z)P_1(z)dz\right]^2 \ge 0.$$

Agar $P(y,y)\ge 0$ va $\frac{\partial}{\partial z}P(y,z) = -P_1(y)P_1(z)$ larni ta’minlay olsak $P(y,z)$ funksiya uchun $\tau_2(y)$ funksiyani nolga tengligi kelib chiqadi. Bundan esa $\nu_2(y)$ funksiyani ham nolga tengligi kelib chiqadi.

Agar $\tau_2(y)=0, \nu_2(y)=0$ bo‘lsa Dalamber formulasida $v(x,y)$ ni $v(x,y)\equiv 0$ ekanligi kelib chiqadi , bu esa bizga yechim yagonaligini keltirib chiqaradi.

Masala yechimining mavjudligi.

Dastlab (5) va (6) shartlarni quyidagicha ko‘rinishga keltirib olamiz:

$$v_1^+(x) = I_1(v_1^-(x)), \qquad (5)$$

$$v_2^+(x) = I_2(v_2^-(x)). \qquad (6)$$

Ω_0 sohada $y \to +0$ da limitga o‘tgan ifodamiz

$$\tau_1^{+\prime\prime}(x) - v_1^+(x) = 0 \qquad (9)$$

va Ω_1 sohada olgan

$$\psi_1'\left(\frac{x}{2}\right) = \tau_1^{-\prime}(x) - v_1^-(x), \quad 0 < x < 1 \qquad (7)$$

munosabatlarimiz hamda $\tau_1^-(0) = \psi_1(0)$, $\tau_1^-(1) = \varphi(0)$ shartlarga asosan quyidagi masalaga kelamiz:

$$\begin{cases} \tau_1^{+\prime\prime}(x) - I_1(\tau_1^{+\prime}(x)) = -I_1\left(\psi_1'\left(\frac{x}{2}\right)\right) \\ \tau_1^+(0) = \psi_1(0), \quad \tau_1^+(1) = \varphi(0). \end{cases}$$

$K(x,z)$ yadroni $K(x,z) = K_1(x)K_1(z)$ deb olsak I_1 operatorni

$$I_1(f(x)) = \int_0^x f(z)K_1(x)K_1(z)dz \qquad (10)$$

ko‘rinishga keladi. Natijada quyidagi integral tenglamaga kelamiz:

$$\tau_1^{+\prime\prime}(x) - \int_0^x \tau_1^{+\prime}(x)K_1(x)K_1(z)dz = \tilde{\psi}_1(x), \qquad (11)$$

bu yerda $\tilde{\psi}_1(x) = -\int_0^x \psi_1'\left(\frac{x}{2}\right)K_1(x)K_1(z)dz$.

Hosil bo‘lgan (11) integral tenglamani $(0,x)$ da integrallab

$$\int_0^x \tau_1^{+\prime\prime}(\xi)d\xi - \int_0^x K_1(\xi)d\xi\int_0^\xi \tau_1^{+\prime}(z)K_1(z)dz = \int_0^x \tilde{\psi}_1(\xi)d\xi$$

ifodaga ega bo‘lamiz va

$$\int_0^x \tau_1''(\xi)d\xi = \tau_1'(x) - \tau_1'(0)$$

ni hisobga olib integrallash tartibini o‘zgartirsak quyidagi integral tenglamaga kelamiz:

$$\tau_1^{+\prime}(x)dx - \int_0^x \tau_1^{+\prime}(z)K_1(z)dz\int_z^x K_1(\xi)d\xi = \tilde{\psi}_1(x) + \tau_1^{+\prime}(0)$$

bu yerda $\tilde{\psi}_1(x) = \int_0^x \tilde{\psi}_1(\xi)d\xi$.

Hozircha $\tau_1'(0)$ ni ma'lum deb olib integral tenglamani yechishda davom etamiz va quyidagi belgilashlarni kiritib olamiz:

$K_1(x,z) = K_1(z)\int_z^x K_1(\xi)d\xi$ - yadro uzluksiz yoki kichik maxsuslikka ega boʻlishi mumkin.

$\tilde{f}(x) = \tilde{\psi}_1(x) + \tau_1'(0)$ esa uzluksiz funksiya boʻlsin.

Bularni hisobga olib integral tenglama yechimini rezolventa orqali quyidagicha yozish mumkin boʻladi:

$$\tau_1'(x) = \tilde{f}(x) + \int_0^x R(x,z)\tilde{f}(z)dz .$$

Bu yerda $R(x,z)$ funksiya $K_1(x,z)$ yadroning rezolventasi.

Ushbu tenglikni $(0,x)$ intervalda integrallab $\tau_1(x)$ funksiyani topamiz.

$$\tau_1(x) - \tau_1(0) = \int_0^x \tilde{f}(\xi)d\xi + \int_0^x\int_0^\xi R(x,z)\tilde{f}(z)dzd\xi$$

yoki

$$\tau_1(x) = \int_0^x \tilde{f}(\xi)d\xi + \int_0^x\int_0^\xi R(x,z)\tilde{f}(z)dzd\xi + \psi_1(0) .$$

Endi yuqorida ma'lum deb olib davom etgan qiymat $\tau_1'(0)$ ni topish uchun quyidagi hisob kitoblarni amalga oshiramiz:

$$\tau_1(x) = \int_0^x \left[\tilde{\psi}_1(\xi) + \tau_1'(0)\right]d\xi + \int_0^x\int_0^\xi R(x,z)\left[\tilde{\psi}_1(z) + \tau_1'(0)\right]dzd\xi + \psi_1(0) \qquad (12)$$

yoki

$$\tau_1(x) = \int_0^x \tilde{\psi}_1(\xi)d\xi + \int_0^x \tau_1'(0)d\xi + \int_0^x\int_0^\xi R(x,z)\tilde{\psi}_1(z)dzd\xi + \int_0^x\int_0^\xi R(x,z)\tau_1'(0)dzd\xi + \psi_1(0) .$$

Ushbu belgilashni $\tilde{F}(x,z) = \int_0^x \tilde{\psi}_1(\xi)d\xi + \int_0^x\int_0^\xi R(x,z)\tilde{\psi}_1(z)dzd\xi + \psi_1(0)$ kiritib olib hisob kitobni sodda koʻrinishda tasvirlashimiz miumkin.

$$\tau_1(x) = \int_0^x \tau_1'(0)d\xi + \int_0^x\int_0^\xi R(x,z)\tau_1'(0)dzd\xi + \tilde{F}(x,z) .$$

$x=1$ dagi $\tau_1(x)$ funksiyani qiymatidan foydalangan holda $\tau_1'(0)$ ni topishimiz mumkin boʻladi:

$$\tau_1(1) = \tau_1'(0) + \tau_1'(0)\int_0^1\int_0^\xi R(1,z)dzd\xi + \tilde{F}(1,z),$$

$$\tau_1'(0) = \frac{\varphi(0) - \tilde{F}(1,z)}{1 + \int_0^1\int_0^\xi R(1,z)dzd\xi}.$$

Bundan $\int_0^1\int_0^\xi R(1,z)dzd\xi \neq -1$ qo‘shimcha shartga ega bo‘lamiz.

$\tau_1(x)$ - (12) ko‘rinishda topilgandan so‘ng (7) tenglikdan foydalanib $\nu_1^+(x)$ ni topish mumkin bo‘ladi.

Ulash shartiga ko‘ra esa $\nu_1^-(x)$ aniq ko‘rinishda topiladi.

Ω_2 sohada $U_{xx} - U_{yy} = 0$ tenglamaning $U(-0,y) = \tau_2^-(y)$, $0 \le y \le 1$, $U_x(-0,y) = \nu_2^-(y)$, $0 < y < 1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Dalamber formulasi orqali yozib olamiz [3].

$$U(x,y) = \frac{1}{2}\left[\tau_2(y-x) + \tau_2(y+x)\right] - \frac{1}{2}\int_{y+x}^{y-x}\nu_2^-(z)dz.$$

Bu yechimni (3) shartga bo‘ysundirib olgan (8) tengligimizga I_2 operatorni qo‘llab

$I_2\left(\tau_2'(y)\right) - I_2\left[\psi_1'\left(\frac{y}{2}\right)\right] = \nu_2^+(y)$ tenglikka ega bo‘lamiz.

Keyin esa

$$\nu_2^+(y) = \int_0^1 \tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi + \tau_2(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\Bigg|_0^y -$$

$$-\int_0^y \tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta - \varphi(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\Bigg|_0^y +$$

$$+\int_0^y \varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta$$

yoki

$$I_2\left(\tau_2{}'(y)\right)-I_2\left[\psi_1'\left(\frac{y}{2}\right)\right]=\int_0^1\tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi+\tau_2(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\Bigg|_0^y-$$

$$-\int_0^y\tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta-\varphi(\eta)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\Bigg|_0^y+$$

$$+\int_0^y\varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta\,.$$

Shundan so‘ng quyidagiga ega bo‘lamiz:

$$I_2\left(\tau_2{}'(y)\right)-I_2\left[\psi_1'\left(\frac{y}{2}\right)\right]=\int_0^1\tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi-\tau_2(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n)}{\sqrt{y}}}-$$

$$-\int_0^y\tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta-\varphi(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n+1)}{\sqrt{y}}}+$$

$$+\int_0^y\varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta.$$

Bu tenglikda ma’lum narsalarni bir tomonga olib o‘tsak

$$I_2\left(\tau_2{}'(y)\right)+\int_0^y\tau_2'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta=\tilde{F}_2(y)\qquad(13)$$

hosil bo‘ladi. Bu yerda

$$\tilde{F}_2(y)=I_2\left[\psi_1'\left(\frac{y}{2}\right)\right]+\int_0^1\tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi-\tau_2(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n)}{\sqrt{y}}}+$$

$$+\varphi(0)\cdot\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y}}e^{-\frac{(2n+1)}{\sqrt{y}}}+\int_0^y\varphi'(\eta)\left(\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n+1)}{\sqrt{y-\eta}}}\right)d\eta$$

I_2 operatorni $I_2\left(g(y)\right)=\int_0^y g(y)P(y,z)dz$ ko‘rinishda tanlab olib, (13) tenglikni quyidagicha yozib olish mumkin:

$$\int_0^y\tau_2'(\eta)\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta=\tilde{F}_2(y).\qquad(14)$$

(14) tenglikni y bo‘yicha differensiallab quyidagi integral tenglamaga kelamiz:

$$\tau_2'(\eta)\left\{\lim_{\eta\to y}\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)\right\}+$$

$$+\int_0^y \tau_2'(\eta)\frac{\partial}{\partial y}\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)d\eta=\tilde{F}_2(y).$$

$\lim_{\eta\to y}\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)=\tilde{P}(y)$ belgilash kiritib, $\tilde{P}(y)\neq 0$ deb olib tenglamani har ikki tomoniga bo'lib yuborsak

$$\tau_2'(\eta)+\int_0^y \tau_2'(\eta)\frac{\frac{\partial}{\partial y}\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)}{\tilde{P}(y)}d\eta=\frac{\tilde{F}_2(y)}{\tilde{P}(y)}$$

hosil bo'ladi.

$$\frac{\frac{\partial}{\partial y}\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)}{\tilde{P}(y)}=Q(y,\eta);\quad \frac{\tilde{F}_2(y)}{\tilde{P}(y)}=\Phi(y);$$

belgilashlarni kiritib olamiz.

$Q(y,\eta)$ - yadro uzluksiz yoki 1 dan kichik maxsuslikka ega, $\Phi(y)$-funksiya esa uzluksiz bo'lsa, $\tau_2'(\eta)+\int_0^y \tau_2'(\eta)Q(y,\eta)d\eta=\Phi(y)$ Volterra 2-tur integral tenglama quyidagi yechimga ega:

$$\tau_2'(\eta)=\Phi(y)+\int_0^y \Phi(y)R_2(y,\eta)d\eta.$$

$R_2(y,\eta)$ - $Q(y,\eta)$ yadroning rezolventasi.

$\tau_2'(y)$ ni (8) tenglikka olib borib qo'yib $\nu_2^-(x)$ ni topish mumkin bo'ladi.

Topilganlarga asosan qo'yilgan {(1),(2),(3),(4),(5)} masala yechimini Ω_0 sohada 1-chegaraviy masala yechimi ko'rinishda Ω_1 va Ω_2 da esa Dalamber formulasi orqali tiklab ko'rsatish mumkin.

Shunday qilib quyidagi tasdiq o'rinli ekanligini isbotladik:

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$$K(x,z)=K_1(x)K_1(z)\,,\ \frac{\partial}{\partial z}K(x,z)=-K_1(x)K_1(z)\,,K(x,x)\geq 0\,,\ \int\limits_0^1\int\limits_0^{\xi}R(1,z)dzd\xi\neq -1$$

$$,\ \frac{\partial}{\partial z}P(y,z)=-P_1(y)P_1(z)\,,\ P(y,y)\geq 0\,,$$

$$\lim_{\eta\to y}\left(P(y,\eta)+\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{y-\eta}}e^{-\frac{(2n)}{\sqrt{y-\eta}}}\right)=P(y)\neq 0\,,\ \ \varphi(y),\psi_i(x)\in C^1[0,1],(i=1,2)\,,$$

$$I_1\left(f(x)\right)=\int\limits_0^x f(z)K(x,z)dz\,,\ I_2\left(g(y)\right)=\int\limits_0^y g(y)P(y,z)dz$$

shartlar bajarilsa, u holda masalaning yechimi mavjud va yagona.

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