Volume-11| Issue-1| 2023 **Research Article**

YUKLANGAN KASR TARTIBLI INTEGRO-DIFFERENSIAL TENGLAMALAR UCHUN MASALALAR

https://doi.org/10.5281/zenodo.7568362

Omonova Dinora Dilshodjon qizi
Farg`ona davlat universiteti talabasi.

Received: 22-01-2023
Accepted: 22-01-2023
Published: 22-01-2023

Abstract: Ushbu maqolada Kaputo kasr tartibli operator qatnashgan yuklangan tenglama uchun masala o`rganilgan. Bu masalalar yechimlari Koshi masalasi yechimdan foydalanib topilgan.

Keywords:. yuklangan integro-differensial tenglama, kasr tartibli operator, Koshi masalasi.

ЗАДАЧИ ДЛЯ ЗАГРУЖЕННЫХ ИНТЕГРО- ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ДРОБНОГО ПОРЯДКА

Received: 22-01-2023
Accepted: 22-01-2023
Published: 22-01-2023

Abstract: В данной статье изучались две задачи для нагруженного уравнения с дробным оператором по Капуто. Найдены решения этих задач с использованием решения задачи Коши..

Keywords:. нагруженное интегро-дифференциальное уравнение, оператор дробного порядка, задача Коши.

About: FARS Publishers has been established with the aim of spreading quality scientific information to the research community throughout the universe. Open Access process eliminates the barriers associated with the older publication models, thus matching up with the rapidity of the twenty-first century.

PROBLEMS FOR LOADED INTEGRO-DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER

Received: 22-01-2023
Accepted: 22-01-2023
Published: 22-01-2023

Abstract: In this article,the problem for the loaded equation involving the Caputo fractional operator is studied.The solution of these problems are found using the solution of the Cauchy problem.

Keywords:. loaded integro-differential equation, fractional order operator, Cauchy problem..

About: FARS Publishers has been established with the aim of spreading quality scientific information to the research community throughout the universe. Open Access process eliminates the barriers associated with the older publication models, thus matching up with the rapidity of the twenty-first century.

I. Kirish. So‘ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug‘ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo‘yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi. Yuklangan xususiy hosilali va oddiy differensial tenglamalar yuklangan differensial tenglama ko`plab tadqiqotchilar tomonidan o‘rganilgan (masalan, ushbu [1]–[3] ishlarga qaralsin).

II. Masalaning qo‘yilishi va tadqiqoti.

(0,1) oraliqda ushbu

$$ {}_cD_{0x}^{\alpha}y(x)-\lambda I_{0x}^{\gamma}y(x)=f(x) \qquad (1)$$

kasr tartibli integro - differensial tenglamani qaraylik, bu yerda $y(x)$- noma'lum funksiya; α,γ,λ - o'zgarmas haqiqiy sonlar bo'lib, $\gamma>0$; ${}_cD_{0x}^{\alpha}y(x)$- Kaputo ma'nosida α (kasr) tartibli hosila operatori, $I_{0x}^{\gamma}y(x)$- Riman-Liuvill ma'nosida γ (kasr) tartibli integral operatori:

$$ {}_cD_{0x}^{\alpha}y(x)=\frac{1}{\Gamma(1-\alpha)}\int_0^x (x-t)^{-\alpha}\, y'(t)\,dt,\ x>0,$$

$$ I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}\, y(t)\,dt,\ x>0.$$

A masala. Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) (0,1) oraliqda (1) tenglamani qanoatlantirsin;

2) $x=0$ nuqtada esa

$$ y(0)=A, \qquad (2)$$

shartni qanoatlantirsin, bu yerda A - berilgan o'zgarmas haqiqiy son.

(1) tenglamaga $I_{0x}^{\alpha}y(x)$ ni ta'sir ettirib,

$$ I_{0x}^{\alpha}\left\{I_{0x}^{\gamma}y(x)\right\}=I_{0x}^{\alpha+\gamma}y(x),\ I_{0x}^{\alpha}\{{}_cD_{0x}^{\alpha}y(x)\}=y(x)-y(0) \qquad (3)$$

(3) xossalardan va y(0)=A shartdan foydalanib,uni quyidagicha yozib olamiz:

$$ y(x)=\lambda I_{0x}^{\alpha+\gamma}y(x)+I_{0x}^{\alpha}f(x)+A \qquad (4)$$

ko'rinishdagi integral tenglamani hosil qilamiz.

(4) Volterra integral tenglamasi bo'lib,

$$ y(x)-\frac{\lambda}{\Gamma(\alpha+\gamma)}\int_0^x (x-z)^{\alpha+\gamma-1}y(z)\,dz=A+\frac{1}{\Gamma(\alpha)}\int_0^x (x-z)^{\alpha-1}f(z)\,dz \qquad (5)$$

uni yechish uchun ba'zi belgilashlarni kiritamiz:

$$ g(x)=\frac{1}{\Gamma(\alpha)}\int_0^x (x-z)^{\alpha-1}f(z)\,dz+A,\ K(x,z)=\frac{(x-z)^{\alpha+\gamma-1}}{\Gamma(\alpha+\gamma)} \qquad (6)$$

(5) tenglamani ketma-ket yaqinlashish usuli orqali yechamiz.

Buning uchun

$$ K_1(x,z)=\frac{(x-z)^{\alpha+\gamma-1}}{\Gamma(\alpha+\gamma)} \quad \text{va} \quad K_i(x,y)=\int_y^x K_1(x,t)K_{i-1}(t,y)\,dt$$

formulalardan foydalanib,ba'zi hisoblashlarni amalga oshirib,

$$ K_n(x,z)=\frac{(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(n(\alpha+\gamma))}$$

ko'rinishda topamiz. $K_n(x,z)$ yadrolarning rezolventasi

$$R(x,z,\lambda)=\sum_{n=1}^{+\infty}\frac{\lambda^{n-1}(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(m(\alpha+\gamma))}$$

ko‘rinishda bo‘ladi.

Integral tenglamalar nazariyasiga ko‘ra (5) tenglamani yechimini,

$$y(x)=g(x)-\lambda\int_0^x R(x,z,\lambda)g(z)dz$$

ko‘rinishda topamiz, bu yerda $R(x,z,\lambda)=\sum_{n=1}^{+\infty}\frac{\lambda^{n-1}(x-z)^{n(\alpha+\gamma)-1}}{\Gamma(m(\alpha+\gamma))}$.

(6) belgilashlarni e’tiborga olib, ba'zi hisoblashlarni amalga oshirib, A masalaning yechimini

$$y(x)=AE_{\alpha+\gamma,1}\left(\lambda x^{\alpha+\gamma}\right)+\int_0^x (x-z)^{\alpha-1} f(z)E_{\alpha+\gamma,\alpha}\lambda(x-z)^{\alpha+\gamma}dz \quad (7)$$

ko'rinishda topamiz, bu yerda $E_{p,q}(z)=\sum_{n=0}^{+\infty}\frac{z^n}{(pn+q)}$ - Mittag-Leffer funksiyasi.

Endi

$$_cD_{0x}^{\alpha}y(x)-\lambda I_{0x}^{\gamma}y(x)=y(x_0) \quad (8)$$

yuklangan tartibli integro-differensial tenglamani qaraylik

B masala. Shunday $y(x)$ funksiya topilsinki, u (8) tenglamani va (2) shartni qanoatlantirsin.

Bu masalaning yechimini A masalaning yechimidan foydalanib,

$$y(x)=AE_{\alpha+\gamma,1}\left(\lambda x^{\alpha+\gamma}\right)+y(x_0)x^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x^{\alpha+\gamma}\right) \quad (9)$$

ko'rinishda aniqlanadi.

(9) dan $x=x_0$ ni o'rniga qo'yib, $y(x_0)$ ni

$$y(x_0)=\frac{AE_{\alpha+\gamma,1}\left(\lambda x_0^{\alpha+\gamma}\right)}{1-x_0^{\alpha}E\left(\lambda x_0^{\alpha+\gamma}\right)} \quad (10)$$

ko`rinishda topamiz.

(10) ni (9) ga olib borib qo'yilib yechim B masalani yechimini

$$y(x)=AE_{\alpha+\gamma,1}\left(\lambda x^{\alpha+\gamma}\right)+\frac{AE_{\alpha+\gamma,1}\left(\lambda x_0^{\alpha+\gamma}\right)}{1-x_0^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x_0^{\alpha+\gamma}\right)}x_0^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x_0^{\alpha+\gamma}\right) \quad (11)$$

ko'rinishda topamiz.

1-teorema. Agar $x_0^{\alpha}E_{\alpha+\gamma,\alpha+1}\left(\lambda x_0^{\alpha+\gamma}\right)\neq 1$ bo‘lsa, u holda B masala yagona yechimga ega bo‘lib, u (11) formula bilan aniqlanadi.

Ta’kidlash joizki B masalaga o`xshash masala [4] ishda ko`rilgan.

FOYDALANILGAN ADABIYOTLAR:

1. Urinov A.K., Azizov M.S. Boun dary value problems for a fourth order partial differential equation with an unknown right-hand part. Lobochevskii Journal of Mathematics, vol. 42, pp.632-640. 2021

2. Tillabayeva G.I. Birinchi tartibli oddiy differensial tenglama uchun nolakal shartli masalalar. NamDU ilmiy axborotnomasi. 2020-yil 1-son 3-6 betlar.

3. Tillabayeva G.I. O'ng tomoni noma'lum va koeffisiyenti uzulishga ega bo'lgan birinchi tartibli chiziqli oddiy differensial tenglama uchun Bitsadze-Samariskiy masalas.i NamDU ilmiy axborotnomasi. 2020-yil 2-son 20-26 betlar.

4. Omonova D.D. Yuklangan kasr tartibli oddiy differensial tenglamalar uchun masalalar. Ijodkor o'qituvchi. Vol. 2 No. 24 (2022). 252- 255- betlar.