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# A GOURSE PROBLEM FOR A FOURTH-ORDER EQUATION WITH TWO SINGULAR COEFFICIENTS

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**Botyrova Khilola Ilhomjon's daughter**
*Master's student of Fardu*

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**Abstract:** In the article, I showed the method of solving the fourth-order hyperbolic equation using the substitution operator. Nowadays, this method is a developing method and is important in mathematical physics. As a result of the calculations in the article, the solution of the equation is significant.
**Keywords:** Bessel operator, Gursa problem, classical solution, singular coefficient,



1. **Introduction**. The problem of finding a classical solution of a fourth-order hyperbolic equation with a singular coefficient using the modern method of mathematical physics - the substitution operator is solved in this article. First, we put the problem, for this we enter it like the field for which the solution is sought. The field consists of a rectangle, and the values of the function are given at the boundaries of the field. During the solution of the problem, we take a fixed point from the inside of the field, using it to find the solution in a transparent way. we will have

**2. Main part**

$\Omega\{(x,y): 0<x<b, 0<y<h\}$ in this field

$$L_0^{\alpha,\beta} B_{x,\alpha-\frac{1}{2}} B_{y,\beta-\frac{1}{2}} U(x,y) = \left(\frac{\partial^2}{\partial x^2}+\frac{2\alpha}{x}\frac{\partial}{\partial x}\right)\left(\frac{\partial^2}{\partial y^2}+\frac{2\beta}{y}\frac{\partial}{\partial y}\right)U =$$

$$= \frac{\partial^4 U}{\partial x^2 \partial y^2}+\frac{2\alpha}{x}\frac{\partial^3 U}{\partial x \partial y^2}+\frac{2\beta}{y}\frac{\partial^3 U}{\partial y \partial x^2}+\frac{4\alpha\beta}{xy}\frac{\partial^2 U}{\partial x \partial y}=0, \qquad (1)$$

let's look at the equation here

$$B_{x,p} \equiv \frac{\partial^2}{\partial x^2}+\frac{2p+1}{x}\frac{\partial}{\partial x} \quad \alpha,\beta,p \in R, 0<\alpha,\ \beta<\frac{1}{2}$$ – Bessel differential operator

The general solution of equation (1) has the following form

$$U(x,y)=\frac{y^{1-2\beta}}{1-2\beta}g_1(x)+g_2(x)+\frac{x^{1-2\alpha}}{(1-2\alpha)(1-2\beta)}\int_0^y [y^{1-2\beta}-t^{1-2\beta}]t^{2\beta}f_1(t)dt$$
$$+\frac{1}{1-2\beta}\int_0^y [y^{1-2\beta}-t^{1-2\beta}]t^{2\beta}f_2(t)dt \qquad (2_1)$$

or

$$U(x,y)=\frac{x^{1-2\alpha}}{1-2\alpha}f_1(y)+f_2(y)+\frac{y^{1-2p}}{(1-2\alpha)(1-2p)}\int_0^x[x^{1-2\alpha}-s^{1-2\beta}]s^{2\alpha}g_1(s)ds$$
$$+\frac{1}{1-2\alpha}\int_0^x[x^{1-2\alpha}-s^{1-2\beta}]s^{2\alpha}g_2(s)ds \qquad (2_2)$$

here $f_k(y), g_k(x), (k=1,2)$ arbitrary continuously differentiable functions.

For equation (1), $\Omega$ we study the analogue of Gursa's problem in the field.

**Gursa issue.** $\Omega$ in the area (1) of this equation

$$U(0,y)=\varphi_1(y),\quad \lim_{x\to+0}x^{2\alpha}U_x(x,y)=\varphi_2(y), 0\le y\le h, \qquad (3)$$

$$U(x,0)=\psi_1(x),\quad \lim_{y\to+0}y^{2\beta}U_y(x,y)=\psi_2(x), 0\le x\le l; \qquad (4)$$

find a classical solution satisfying the conditions here $\varphi_k(y), \psi_k(x)\,(k=1,2)$ given functions.

To solve this problem, the general solution (21) is optional by putting conditions (3) and (4) $f_k(y), g_k(x), (k=1,2)$ we find functions.

$$f_1(y)=y^{-2p}[y^{2p}\varphi_2'(y)]',\quad f_2(y)=y^{-2\beta}[y^{2\beta}\varphi_1'(y)]',$$
$$g_2(x)=\psi_1(x)\quad g_1(x)=\psi_2(x)$$

Found $f_k(y), g_k(x), (k=1,2)$ putting ($2_1$) into the general solution,

$$U(x,y)=\psi_1(x)+\frac{y^{1-2\beta}}{1-2\beta}\psi_2(x)+\frac{x^{1-2\alpha}}{(1-2\alpha)(1-2\beta)}\int_0^y[y^{1-2p}-t^{1-2\beta}]t^{2\beta}t^{-2\beta}[t^{2p}\varphi_2'(t)]'dt+$$
$$+\frac{1}{1-2\beta}\int_0^y[y^{1-2\beta}-t^{1-2p}]t^{2p}t^{-2p}[t^{2p}\varphi_1'(t)]'dt$$

we create a solution.. $\varphi_1(y)$ and $\varphi_2(y)$ xosilalari qatnashgan integrallarini hisoblaymiz.

We calculate the integrals involving derivatives

$$U(x,y)=\varphi_1(y)+\psi_1(x)+\frac{y^{1-2\beta}}{1-2\beta}\psi_2(x)+\frac{x^{1-2\alpha}}{(1-2\alpha)}[\varphi_2(y)-\varphi_2(0)]-\varphi_1(0)$$

**Teorema.** If $0<\alpha,\ \beta<\frac{1}{2}$, $\varphi_k(y)\in C[0,h]\cap C^2(0,h),\ \psi_k(x)\in C[0,l]\cap C^2(0,l)$, be

$\varphi_k(y)$ function $y\to+0$ in $2\beta$ from small and $\psi_k(x)$ function $x\to+0$ in $2\alpha$ from may have a small specialty.

Then this

$$U(x,y)=\varphi_1(y)+\psi_1(x)+\frac{y^{1-2\beta}}{1-2\beta}\psi_2(x)+\frac{x^{1-2\alpha}}{(1-2\alpha)}[\varphi_2(y)-\varphi_2(0)]-\varphi_1(0) \qquad (5)$$

function is the only solution to the Gursa problem.

To prove the theorem, it is necessary to show that function (5) satisfies equation (1) and conditions (3), (4).

For this, we check that function (5) satisfies conditions (3) and (4). $\varphi_2(0)=0$ from equality $U(0,y)=\varphi_1(y)$, $\lim_{x\to 0}x^{2\alpha}U_x=\varphi_2(y)-\varphi_2(0)=\varphi_2(y)$, it follows that the conditions are appropriate. $\varphi_2(0)=0$, $\psi_2(0)=0$ tengliklardan foydalanib $U(x,0)=\varphi_1(0)+\psi_1(x)+\frac{x^{1-2\alpha}}{1-2\alpha}[\varphi_2(0)-\varphi_2(0)]-\varphi_1(0)$, $U(x,0)=\psi_1(x)$ $\lim_{y\to o} y^{2\beta}U_y=\psi_2(x)$

We show that the conditions are appropriate. We take the necessary derivatives from the function (5) and check that it satisfies the equation (1).

$$U_x(x,y)=\psi_1'(x)+x^{-2\alpha}\varphi_2(y)+\frac{y^{1-2\beta}}{1-2\beta}\psi_2'(x)$$

$$U_{xxy}(x,y)=-2\alpha x^{-2\alpha-1}\varphi_2'(y)+y^{-2\beta}\psi_2''(x)$$

$$U_{xxyy}(x,y)=-2\alpha x^{-2\alpha-1}\varphi_2''(y)-2\beta y^{-2\beta-1}\psi_2''(x)$$

$$U_y(x,y)=\varphi_1'(y)+\frac{x^{1-2\alpha}}{1-2\alpha}\varphi_2'(y)+y^{-2\beta}\psi_2(x)$$

$$U_{yy}(x,y)=\varphi_1''(y)+\frac{x^{1-2\alpha}}{1-2\alpha}\varphi_2''(y)-2\beta y^{-2\beta-1}\psi_2(x)$$

$$U_{yyx}(x,y)=x^{-2\alpha}\varphi_2''(y)-2\beta y^{-2\beta-1}\psi_2'(x)$$

$$U_{xy}(x,y)=x^{-2\alpha}\varphi_2'(y)-2\beta y^{-2\beta-1}\psi_2'(x)$$

$$U_{xxyy}+\frac{2\alpha}{x}U_{xyy}+\frac{2\beta}{y}U_{yxx}+\frac{4\alpha\beta}{xy}=-2\alpha x^{-2\alpha-1}\varphi_2''(y)-2\beta y^{-2\beta-1}\psi_2''(x)+$$
$$+2\alpha x^{-2\alpha-1}\varphi_2''(y)-4\beta x^{-1}y^{-2\beta-1}\psi_2'(x)-4\alpha\beta y^{-1}x^{-2\alpha-1}\varphi_2'(y)+2\beta y^{-2\beta}\psi_2''(x)+$$
$$+4\alpha\beta x^{-2\alpha-1}y^{-1}\varphi_2'(y)+4\alpha\beta x^{-1}y^{-2\beta-1}\psi_2'(x)=0$$

the theorem is proved

**V. Summary**

An analogue of the Gursa problem was studied for this third-order pseudohyperbolic equation with singular coefficients. It is shown that the solution of the Gursa problem is equivalent to the Volterra integral equation of the second kind. In the second paragraph, the solution to the Gursa issue is clearly found. The property of the fractional Erdey-Kober operator being a substitution operator was used to find the solution. In this case, the solution of the auxiliary problem to the Gursa problem was brought to the studied Gursa problem using the property of the Erdey-Kober operator, and the solution of the problem was found using this solution. After that, the solution of the main problem was found using the solution of the auxiliary problem.

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