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KAPUTO-FABRITSIO OPERATORI ISHTIROK ETGAN DIFFERENSIAL TENGLAMA UCHUN NOLOKAL MASALA

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Abstract: Ushbu ishda vaqt o'zgaruvchisi bo'yicha Kaputo-Fabritsio operatori, fazoviy o'zgaruvchi bo`yicha esa Lejandr operatori qatnashgan xususiy hosilali differensial tenglama uchun nolokal shartli masalaning bir qiymatli yechilishi isbotlangan. Bunda Lejandr polinomlarining xossalaridan foydalanilgan

Keywords: Kaputo-Fabritsio operatori; nolokalshart; Lejandr polinomlari; Furye usuli.

NONLOCAL PROBLEM DIFFERENTIAL EQUATION WITH CAPUTO-FABRIZIO OPERATOR

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Abstract: In this work a unique solvability of nonlocal boundary problem for partial differential equation involving Caputo-Fabrizio operator in time and Legendre operator in space-variable. Properties of Legendre polynomials have been used.

Keywords: Caputo-Fabrizio operator; nonlocal condition; Legendre polynomials; Fourier method.

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Diffuziya jarayonini aks ettiruvchi xususiy hosilali differensial tenglamalar uchun chegaraviy masalalarning bir qiymatli yechilishi ko'plab tadqiqotlarda isbotlangan [1-4].

Ushbu ishda oldingi ishlardan farqli ravishda vaqt o'zgaruvchisi bo'yicha Kaputo-Fabritsio integro-differensial operatori , fazoviy o'zgaruvchisi bo'yicha esa Lejandr operatori qo'llanilgan xususiy hosilali integro-differensial tenglama uchun nolokal shartli masala tadqiq etiladi. Bu operator qatnashgan tenglama uchun lokal masalalar [5] ishda tadqiq etilgan.

Masala. $\Omega=\{(x,t): 0<x<1, 0<t<T\}$ sohada

$$ {}^{CF}D_{0t}^{\alpha}U(x,t)=[(1-x^2)U_x]_x+f(x,t) \qquad (1)$$

tenglama va

$U(x,0) = U(x,\xi) + \varphi(x)$, $-1 \le x \le 1$, $0 \le \xi \le T$ (2)

nolokal shartni qanoatlantiruvchi, hamda $x=-1$, $x=1$ da $U(x,t), U_x(x,t)$ chegaralangan $U(x,t)$ funksiyani toping. Bu yerda

$$^{CF}D_{0t}^{\alpha} g(t) = \frac{1}{1-\alpha}\int_0^t g'(s) e^{\frac{-\alpha}{1-\alpha}(t-s)} ds \qquad (3)$$

$-0 < \alpha < 1$ tartibli Kaputo-Fabritsio operatori [6].

Dastlab $f(x,t) \equiv 0$ bo‘lgan holda (1) tenglamaning yechimini $U(x,t) = \upsilon(t)\cdot X(x) \ne 0$ ko‘rinishda qidiramiz. U holda (1) dan

$$^{CF}D_{0t}^{\alpha}\upsilon(t)\cdot X(x) = [(1-x^2)X''(x) - 2xX'(x)]\upsilon(t)$$

ni olamiz. Bu tenglikni $\upsilon(t)\cdot X(x)$ ga bo‘lsak

$$\frac{^{CF}D_{0t}^{\alpha}\upsilon(t)}{\upsilon(t)} = \frac{[(1-x^2)X''(x) - 2xX'(x)]}{X(x)} = -\lambda.$$

yoki

$$^{CF}D_{0t}^{\alpha}\upsilon(t) + \lambda\upsilon(t) = 0, \qquad (4)$$

$$(1-x^2)X''(x) - 2xX'(x) + \lambda X(x) = 0 \qquad (5)$$

tenglamalar kelib chiqadi. $U(x,t), U_x(x,t)$ funksiyalarning $x=-1$, $x=1$ da chegaralangan yechimga ega ekanligi ma’lum hamda bu yechim

$$X(x) = P_n(x) = \frac{1}{2^n\cdot n!}\cdot\frac{d^n(x^2-1)^n}{dx^n} \quad (n = 0,1,2,...) \qquad (6)$$

Lejandr polinomlari orqali ifodalanadi [7].

Ma’lumki [7], $P_n(x)$ $(n = 0,1,2,...)$ Lejandr polinomlari [-1;1] da ortogonal sistema tashkil qiladi va $\| P_n(x)\|^2 = \frac{2}{2n+1}$.

Ihtiyoriy [-1;1] da bo‘lakli uzlukli $g(x)$ funksiyani $\{P_n(x)\}$ sistema bo‘yicha Furye qatoriga yoyish mumkin [7]:

$$g(x) = \sum_{n=0}^{\infty} C_n P_n(x),\; C_n = \frac{(g, P_n)}{\| P_n\|^2} = \frac{2n+1}{2}\int_{-1}^{1} g(x) P_n(x) dx.$$

Bunday qatorlarni Furye-Lejandr qatorlari deyiladi.

Masala yechimini Furye-Lejandr qatori ko‘rinishida qidiramiz:

$$u(x,t) = \sum_{n=0}^{\infty}\upsilon_n(t) P_n(x), \qquad (7)$$

bu yerda $\upsilon_n(t)$ hozircha noma’lum funksiyalar.

Berilgan funksiyalar $f(x,t)$, $\varphi(x)$ larni ham Furye-Lejandr qatoriga yoyamiz:

$$f(x,t) = \sum_{n=0}^{\infty} f_n(t) P_n(x), \qquad (8)$$

$$\varphi(x)=\sum_{n=0}^{\infty}\varphi_n P_n(x), \qquad (9)$$

bu yerda

$$f_n(t)=\frac{2n+1}{2}\int_{-1}^{1} f(x,t)P_n(x)dx, \quad \varphi_n=\frac{2n+1}{2}\int_{-1}^{1}\varphi(x)P_n(x)dx.$$

(7), (8) ni (1), (2) ga qo‘ysak t o‘zgaruvchiga nisbatan quyidagi masalani olamiz:

$$\begin{cases} {}^{CF}D_{0t}^{\alpha}\upsilon_n(t)+n(n+1)\upsilon_n(t)=f_n(t), \\ \upsilon_n(0)=\upsilon_n(\xi)+\varphi_n. \end{cases} \qquad (10)$$

(10) masalaning yechimini topish uchun [5] da ko‘rilgan Koshi masalasi yechimidan foydalaniladi. Buning uchun $f_n(0)=n(n+1)\upsilon_n(0)$ shart bajarilishi talab etiladi, shunda

$$\upsilon_n(t)=\frac{1-\alpha}{1+n(n+1)(1-\alpha)}f_n(t)+\frac{\alpha}{\left(1+n(n+1)(1-\alpha)\right)^2}\int_0^t f_n(\xi)e^{\frac{-n(n+1)\alpha}{1+n(n+1)(1-\alpha)}(t-\xi)}d\xi+$$
$$\frac{\upsilon_n(0)}{1+n(n+1)(1-\alpha)}e^{\frac{-n(n+1)\alpha}{1+n(n+1)(1-\alpha)}t}$$

(11)

ifoda olinadi. Bundan $\upsilon_n(\xi)$ ni topib, so‘ngra $\upsilon_n(0)=\upsilon_n(\xi)+\varphi_n$ shartga ko‘ra (11) dagi noma’lum o‘zgarmas $\upsilon_n(0)$ ni quyidagicha topamiz:

$$\upsilon_n(0)=\frac{1}{1+n(n+1)(1-\alpha)-e^{\frac{-n(n+1)\alpha}{1+n(n+1)(1-\alpha)}\xi}}\Bigg[\left(1+n(n+1)(1-\alpha)\right)\varphi_n+(1-\alpha)f_n(\xi)-$$
$$-\frac{\alpha}{1+n(n+1)(1-\alpha)}\int_0^{\xi} f_n(s)e^{\frac{-n(n+1)\alpha}{1+n(n+1)(1-\alpha)}(\xi-s)}ds\Bigg].$$

(12)

(7) cheksiz qatorning tekis yaqinlashishini isbotlash uchun dastlab (12) ifoda uchun, so‘ngra esa (11) ifoda uchun baho olamiz:

$$\left|\upsilon_n(0)\right|\le\left|\varphi_n\right|+\frac{C_1\left|f_n(\xi)\right|}{1+n(n+1)(1-\alpha)},$$

$$\left|\upsilon_n(t)\right|\le\frac{\left|\varphi_n\right|}{1+n(n+1)(1-\alpha)}+\frac{C_2\left|f_n(\xi)\right|}{\left(1+n(n+1)(1-\alpha)\right)^2}+\frac{\left|f_n(t)\right|}{1+n(n+1)(1-\alpha)}.$$

Ushbu baholar (7) cheksiz qatorning tekis yaqinlashishini isbotlashda unga majorant bo‘lgan qatorning yaqinlashishini isbotlash uchun muhim ahamiyat kasb etadi. Haqiqatdan ham, agar $\varphi(x), f(x,t)$ funksiyalar o‘z aniqlanish sohasida uzluksiz bo‘lsa

$$\left|u\left(x,t\right)\right| \le \sum_{n=0}^{\infty}\left|\upsilon_n(t)\right|\left|P_n\left(x\right)\right| \le \sum_{n=0}^{\infty}\frac{C}{n^2} < \infty$$

baho o‘rinli. Tenglamada ishtirok etadigan $u_x, u_{xx}, {}^{CF}D_{0t}^{\alpha}u$ funksiyalarga mos keluvchi cheksiz qatorlarning tekis yaqinlashishini isbotlashda berilgan $\varphi(x), f\left(x,t\right)$ funksiyalarga ko‘proq shart tushishi tabiiy. Bunda [8] da ko‘rsatilganidek hisoblashlar asosida $\left|g_n\right| \le \frac{4\sqrt{2}}{\left(2n-3\right)^{3/2}}\left\|g\right\|$ tengsizliklardan foydalansak, berilgan funksiyalarning uzluksizlikdan tashqari, x o‘zgaruvchi bo‘yicha 2-tartibli hosilalari kvadrati bilan jamlanuvchi bo‘lishi sharti ham qo‘yiladi. Masala yechimi yagonaligi esa Furye-Lejandr qatorlari nazariyasiga ko‘ra Lejandr polinomlarining to‘la ortonormal Sistema tashkil qilishidan foydalanib isbotlanadi. Demak, quyidagi tasdiq o‘rinli:

Teorema. *Agar $\varphi(x), f\left(x,t\right)$ funksiyalar o‘z aniqlanish sohasida uzluksiz va x o‘zgaruvchi bo‘yicha 2-tartibli hosilalari kvadrati bilan jamlanuvchi bo‘lsa hamda*

$$f_n\left(0\right) = \frac{n\left(n+1\right)}{1+n\left(n+1\right)\left(1-\alpha\right)-e^{\frac{-n(n+1)\alpha}{1+n(n+1)(1-\alpha)}\xi}}\left[\left(1+n\left(n+1\right)\left(1-\alpha\right)\right)\varphi_n+\left(1-\alpha\right)f_n\left(\xi\right)-\right.$$

$$\left.-\frac{\alpha}{1+n\left(n+1\right)\left(1-\alpha\right)}\int_0^{\xi} f_n\left(s\right)e^{\frac{-n(n+1)\alpha}{1+n(n+1)(1-\alpha)}(\xi-s)}ds\right]$$

shart bajarilsa, u holda masalaning yagona yechimi mavjud va u (7) ko‘rinishda ifodalanadi.

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